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A Transient Thermal Analog of Nuclear Reactors

James R. Hume

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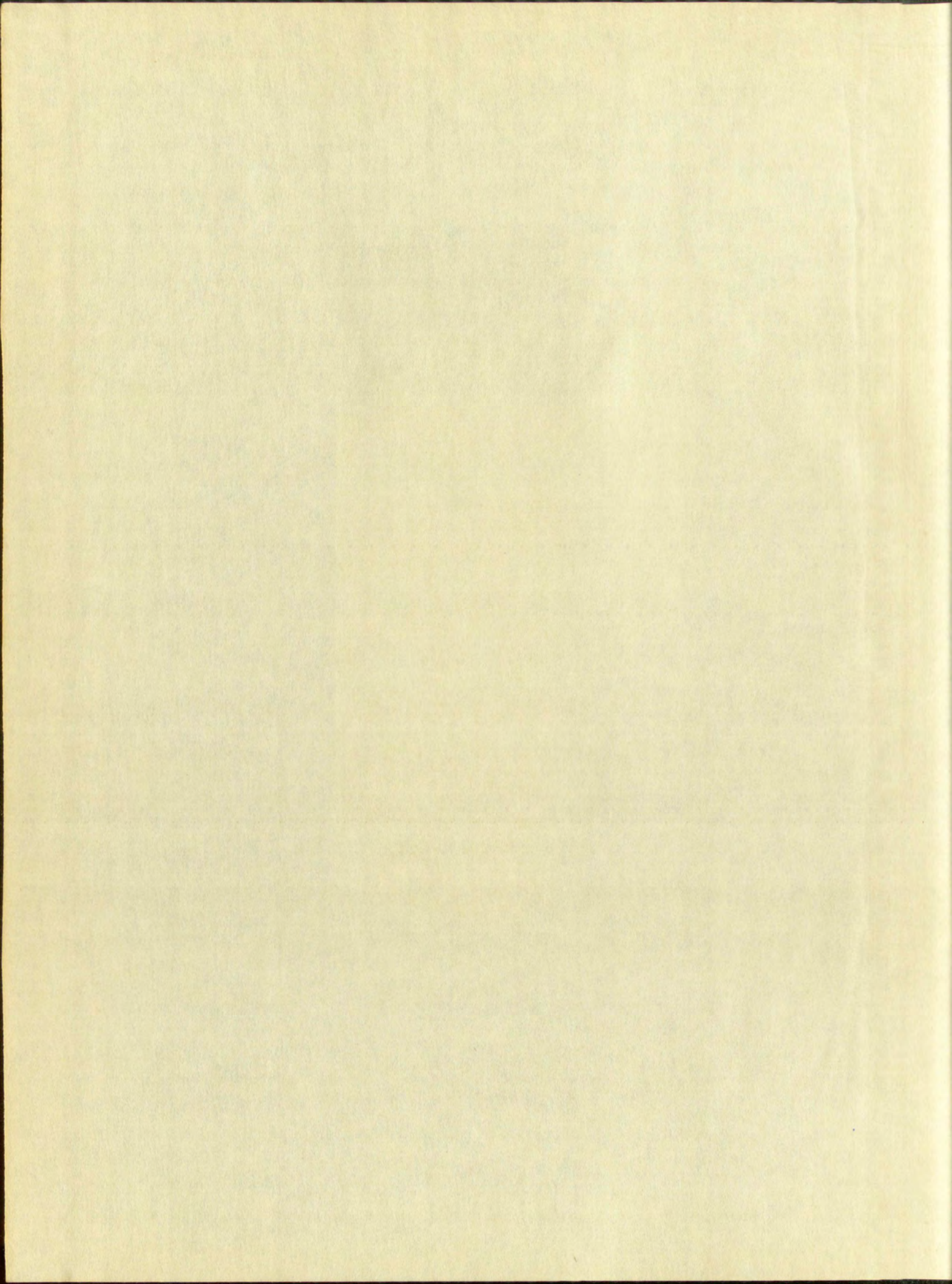
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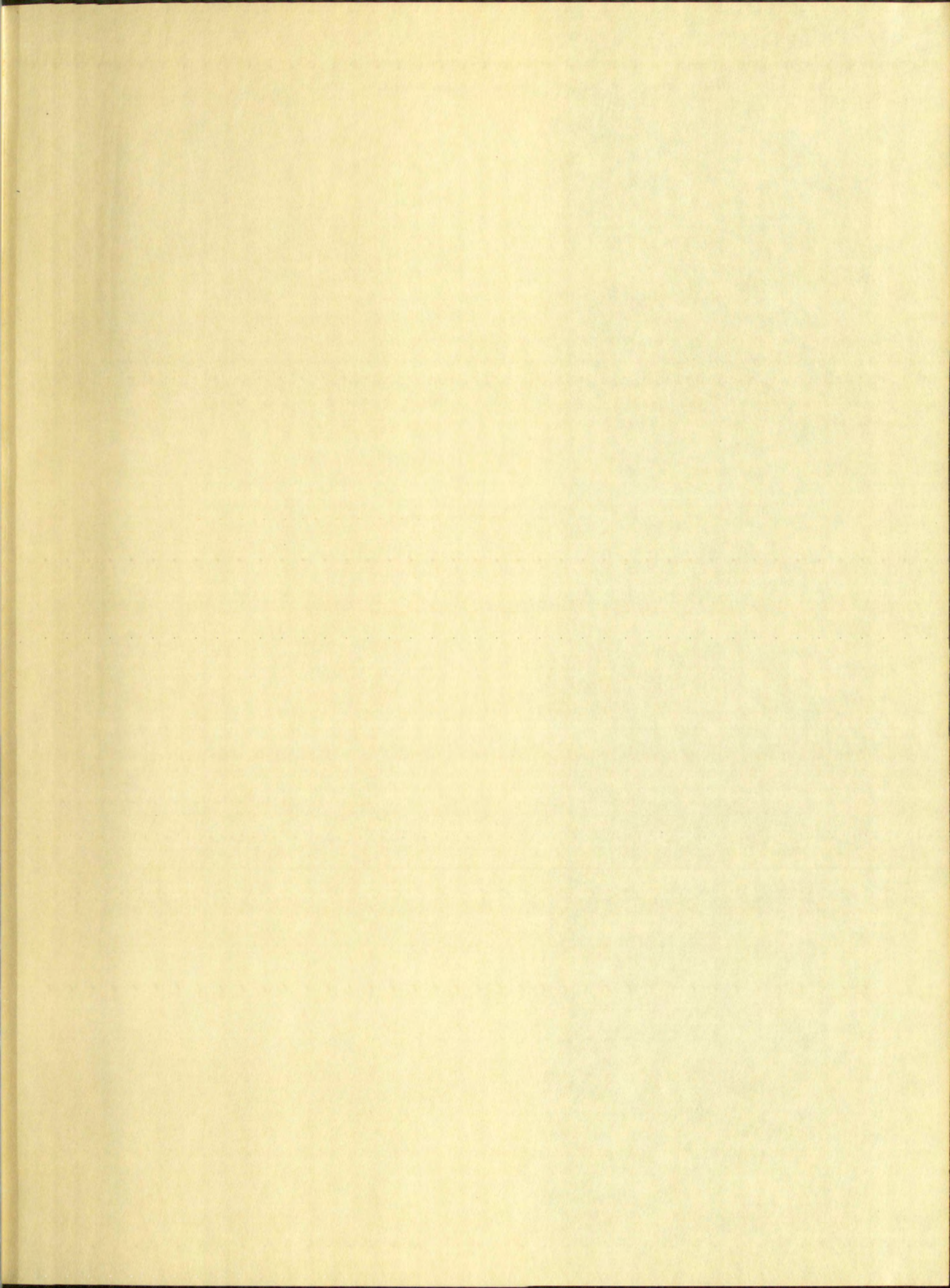
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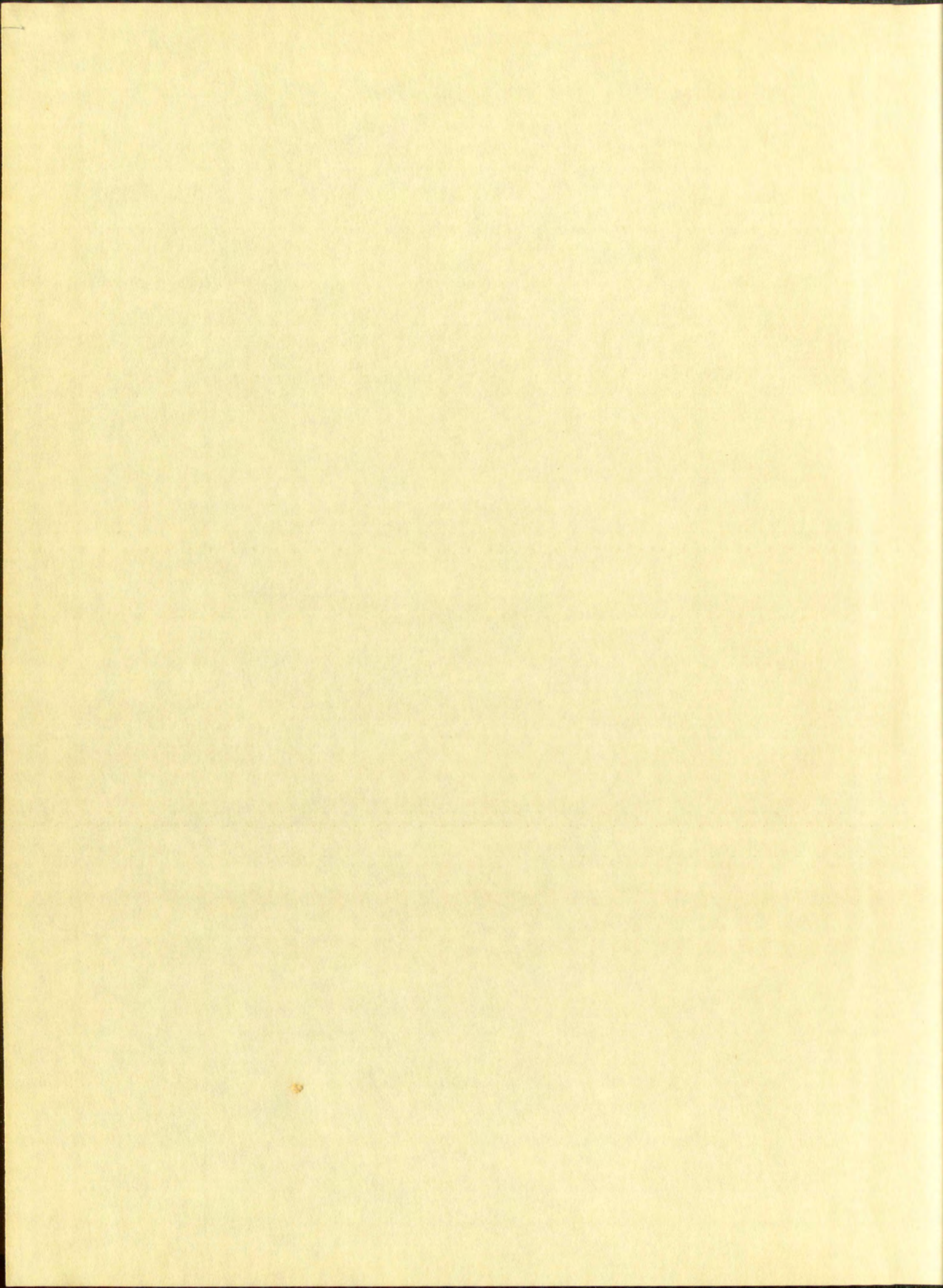
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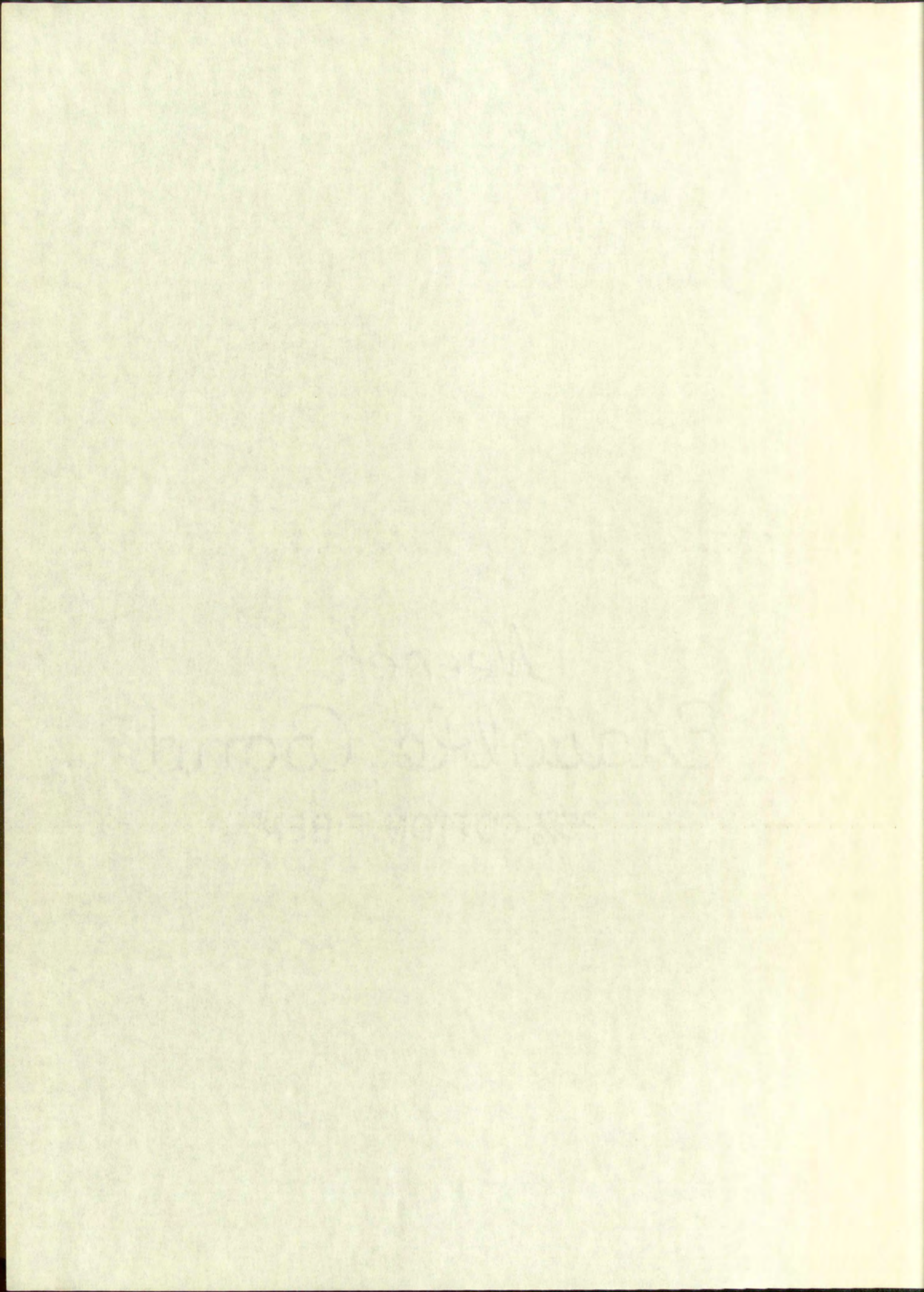
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A TRANSIENT THERMAL ANALOG OF
NUCLEAR REACTORS



By
James R. Hume

A Thesis
Submitted in Partial Fulfillment of the
Requirements for the Degree of
Master of Science in Engineering

The University of New Mexico
1958



A THESIS SUBMITTED TO THE FACULTY OF ENGINEERING
IN CANDIDACY FOR THE DEGREE OF
MASTER OF SCIENCE

BY
JAMES H. HARRIS

A THESIS

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THE UNIVERSITY OF TORONTO

1951

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TABLE OF CONTENTS

CHAPTER	PAGE
1. INTRODUCTION	1
2. LITERATURE REVIEW	4
3. THEORETICAL INVESTIGATION	7
4. EXPERIMENTAL INVESTIGATION	26
5. RESULTS	43
6. CONCLUSIONS AND RECOMMENDATIONS	56
APPENDIX	58

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1 INTRODUCTION
2 LITERATURE REVIEW
3 THEORETICAL FRAMEWORK
4 EMPIRICAL INVESTIGATION
5 RESULTS
6 CONCLUSIONS AND RECOMMENDATIONS
APPENDIX

LIST OF NOMENCLATURE

- x = distance in direction of coolant flow
 t = time
 U_g = steady power, BTU/hr
 ΔU_g = step increase in power, BTU/hr
 h = heat transfer coefficient, BTU/f² hr °F
 p = perimeter of fuel element, f
 m = fuel element temperature, °F
 f = bulk coolant temperature, °F
 c = specific heat, BTU/lb °F
 δ = specific weight, lb/f³
 A = cross-sectional area, f²
 w = fluid flow rate, lb/hr
 X = similarity number, non-dimensional distance
 T = similarity number, non-dimensional time
 M = similarity number, non-dimensional metal temperature
 F = similarity number, non-dimensional fluid temperature
 a = similarity number, ratio of heat capacities
 $b = 1 + 1/a$
 $\bar{F} = \bar{F}(X, p)$, where p is complex Laplace variable
 $\bar{M} = \bar{M}(X, p)$, where p is complex Laplace variable
 V = fluid velocity, f/hr
 z = variable of integration

LIST OF SYMBOLS

- z = distance in direction of coolant flow
- t = time
- g = gravity power, W/m^2
- AD_c = heat transfer in cover, W/m^2
- h = heat transfer coefficient, $W/m^2 \cdot ^\circ C$
- p = pressure of fuel element, Pa
- T_c = fuel element temperature, $^\circ C$
- T_c = bulk coolant temperature, $^\circ C$
- v = specific heat, $J/kg \cdot ^\circ C$
- G = specific weight, N/m^3
- A = cross-sectional area, m^2
- v = fluid flow rate, m^3/s
- X = similarity number, non-dimensional distance
- T = similarity number, non-dimensional time
- M = similarity number, non-dimensional mass
- temperature
- F = similarity number, non-dimensional field
- temperature
- a = similarity number, ratio of heat capacities
- $b = 1 + \lambda v$
- $\beta = \beta(x, y)$, where β is complex Laplace variable
- $\tilde{h} = \tilde{h}(x, y)$, where \tilde{h} is complex Laplace variable
- v = fluid velocity, m/s
- z = variable of integration

D = inside diameter of circular coolant flow channel,
effective diameter of annular coolant flow
channel, f

k = thermal conductivity, $\text{BTU}/f \text{ hr } ^\circ\text{F}$

G = mass velocity = w/A

μ = dynamic viscosity, $\text{lb hr}/f^2$

K = conversion factor for galvanometer trace
deflection, $^\circ\text{F}/\text{in}$

W = weight of thermocouple

Subscripts m and f refer to metal and fluid properties.

U = initial velocity of particles, assumed to be zero
u = effective velocity of particles, assumed to be zero
v = velocity of particles
w = thermal conductivity, W/m² K
x = mass velocity, kg/m² s
y = dynamic viscosity, Pa s
z = conversion factor for dimensional analysis
α = diffusion coefficient, m² s⁻¹
β = weight of particles

Subscripts are used to refer to metal and fluid properties

LIST OF TABLES

TABLE		PAGE
1	Comparison of Thermometer in Special Well and Tube Thermocouples	62
2	Factor K for Converting Oscillograph Trace Deflection to Temperature	64
3	External Heat Loss Measurements	65
4	Raw Data - Metal Temperatures	66
5	Raw Data - Fluid Temperatures	72
6	Results - Metal Temperatures	74
7	Results - Fluid Temperatures	79

TABLE

1	Comparison of the results of the present study with those of other workers
2	and their interpretation
3	Tables of the observed frequencies of the various types of cells
4	Details of the methods used in the present study
5	Material and Methods
6	Results - General Observations
7	Results - Detailed Observations

LIST OF FIGURES

FIGURE		PAGE
1	Differential Cross-section of a Fuel Element and its Cooling Channel	7
2	Overall View of Heated Tube	27
3	Detail of Electrode and Thermocouples.	28
4	Associated Instrumentation	29
5	Summary of Metal Temperature from Numerical Solution	44
6	Non-Dimensional Temperature versus Time for Tests 4, 5, 6	45
7	Non-Dimensional Temperature versus Time for Tests 4, 5, 6	46
8	Non-Dimensional Temperature versus Time for Tests 4, 5, 6	47
9	Summary of Metal Temperatures for Test 1	48
10	Summary of Metal Temperatures for Test 2	49
11	Summary of Metal Temperatures for Test 3	50
12	Fluid Temperature for Tests 1 and 6	51
13	Cross-plot of M versus X from Tests 1, 2, 3	52
14	Electrical Power Circuit	80
15	Typical Portion of Thermocouple Switching Circuit	81
16	Calibration Circuit Schematic	82
17	Thermocouple Current Amplifier	83

CONTENTS

	PAGE
1. Introduction	1
2. General View of the Subject	2
3. Details of the Subject	3
4. Associated Experiments	4
5. Summary of the Subject	5
6. Solution	6
7. Non-Dimensional Representation of the Data	7
8. Tests # 1, 2, 3	8
9. Non-Dimensional Representation of the Data	9
10. Tests # 4, 5, 6	10
11. Non-Dimensional Representation of the Data	11
12. Tests # 7, 8	12
13. Summary of the Subject	13
14. Summary of the Subject	14
15. Summary of the Subject	15
16. Field Experiments for Tests # 1, 2, 3	16
17. Comparison of the Data with the Theory	17
18. Electrical Power Loss	18
19. Typical Form of the Graphical Results	19
20. Circuit	20
21. Calibration of the Instruments	21
22. Thermodynamic Properties of the Gas	22

FIGURE		PAGE
18	Typical Galvanometer Calibration and Thermocouple e.m.f.	84
19	Metal Temperature versus External Heat Loss	85
20	Data Smoothing for Test No. 2	86

Typical ...	18
... ..	19
... ..	20

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Invaluable services were performed by the author's wife, Mary L. Hume, who provided continual encouragement and performed a difficult job of typing to make this paper possible.

JRH

The first part of the paper is devoted to a general discussion of the problem. It is shown that the problem is equivalent to a certain type of boundary value problem for a second order elliptic equation. The second part of the paper is devoted to the construction of a certain type of integral equation. It is shown that this integral equation is equivalent to the original problem. The third part of the paper is devoted to the study of the properties of the integral equation. It is shown that the integral equation has a unique solution. The fourth part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is continuous and satisfies certain boundary conditions. The fifth part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is unique and satisfies certain boundary conditions. The sixth part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is unique and satisfies certain boundary conditions. The seventh part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is unique and satisfies certain boundary conditions. The eighth part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is unique and satisfies certain boundary conditions. The ninth part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is unique and satisfies certain boundary conditions. The tenth part of the paper is devoted to the study of the properties of the solution. It is shown that the solution is unique and satisfies certain boundary conditions.

CHAPTER 1

INTRODUCTION

The nuclear reactor is presently receiving considerable attention as a possible source of energy for stationary power plants and propulsion systems. This has stimulated interest in many engineering problems which become very significant in reactor design work. The response of the reactor core heat transfer system to changes in power generation is one of these problems.

In the reactor, steady power level operation is achieved when the chain reaction is just self-sustaining, i.e. the reproduction of neutrons is exactly one. The expression "reactivity" is defined as the percentage by which the actual reproduction of neutrons differs from one. Positive changes in reactivity can result from many factors such as removal of neutron poisons, insertions of fissioning, moderating, or reflecting materials, and as a result of local temperature changes. Changes such as these occur from start-up and shut-down procedures, changing power demands, and accidents. The effect of positive reactivity is to put the reactor on an approximate exponential power rise which continues until the excess reactivity is removed by some means. Under certain conditions the rate of power increase may be quite high, hence the nuclear reactor has the potential for very large and rapid thermodynamic transients.

Knowledge of the transient response of the reactor core heat transfer system is important design information for control system specification, determination of maximum operating

The first part of the paper deals with the general principles of the theory of the quantum theory of the radiation field. It is shown that the quantum theory of the radiation field is a special case of the general theory of the quantum theory of the radiation field. In the second part of the paper the quantum theory of the radiation field is applied to the theory of the quantum theory of the radiation field. It is shown that the quantum theory of the radiation field is a special case of the general theory of the quantum theory of the radiation field. In the third part of the paper the quantum theory of the radiation field is applied to the theory of the quantum theory of the radiation field. It is shown that the quantum theory of the radiation field is a special case of the general theory of the quantum theory of the radiation field.

temperatures, analysis of thermal stresses, and complete nuclear analysis.

The typical gas-cooled reactor consists of a large mass of neutron moderating material perforated by channels which extend through the assembly. Centered in the channels are the fuel elements. The coolant gas, which is usually pressurized, flows in the channel between fuel and moderator. It is usual to assume that 90% of the heat produced appears in the fuel and 10% in the moderator. The intensity of heat generation is a function of position in the reactor and may be described by trigonometric or Bessel functions depending on the core geometry.

THE INVESTIGATION

The problem of this investigation is the transient response of the bulk coolant temperature and the fuel temperature of a nuclear reactor as a result of a power excursion. The problem is attacked both theoretically and experimentally.

The theoretical portion includes a similarity analysis of an idealized mathematical model of the fuel and coolant channel. This analysis leads to non-dimensional differential equations for which an analytical solution was obtained. Because this solution is so complex, the numerical solution of another investigator is used for comparison with measurements.

The experimental investigations result in measured temperatures produced by a thermal analog of a reactor fuel and coolant channel. The time variation of the metal temperature was

These results are compared with the theoretical model and the experimental data.

The typical gas-liquid reactor consists of a large number of narrow tubular vessels packed in channels which extend through the bed. The gas and liquid flow through the channels in the same direction. The gas is usually preheated in the channel between fuel and oxidizer. It is usual to assume that 90% of the heat produced appears in the fuel and 10% in the oxidizer. The intensity of heat generation is a function of position in the reactor and may be described by trigonometric or Bessel functions depending on the core geometry.

THE INVESTIGATION

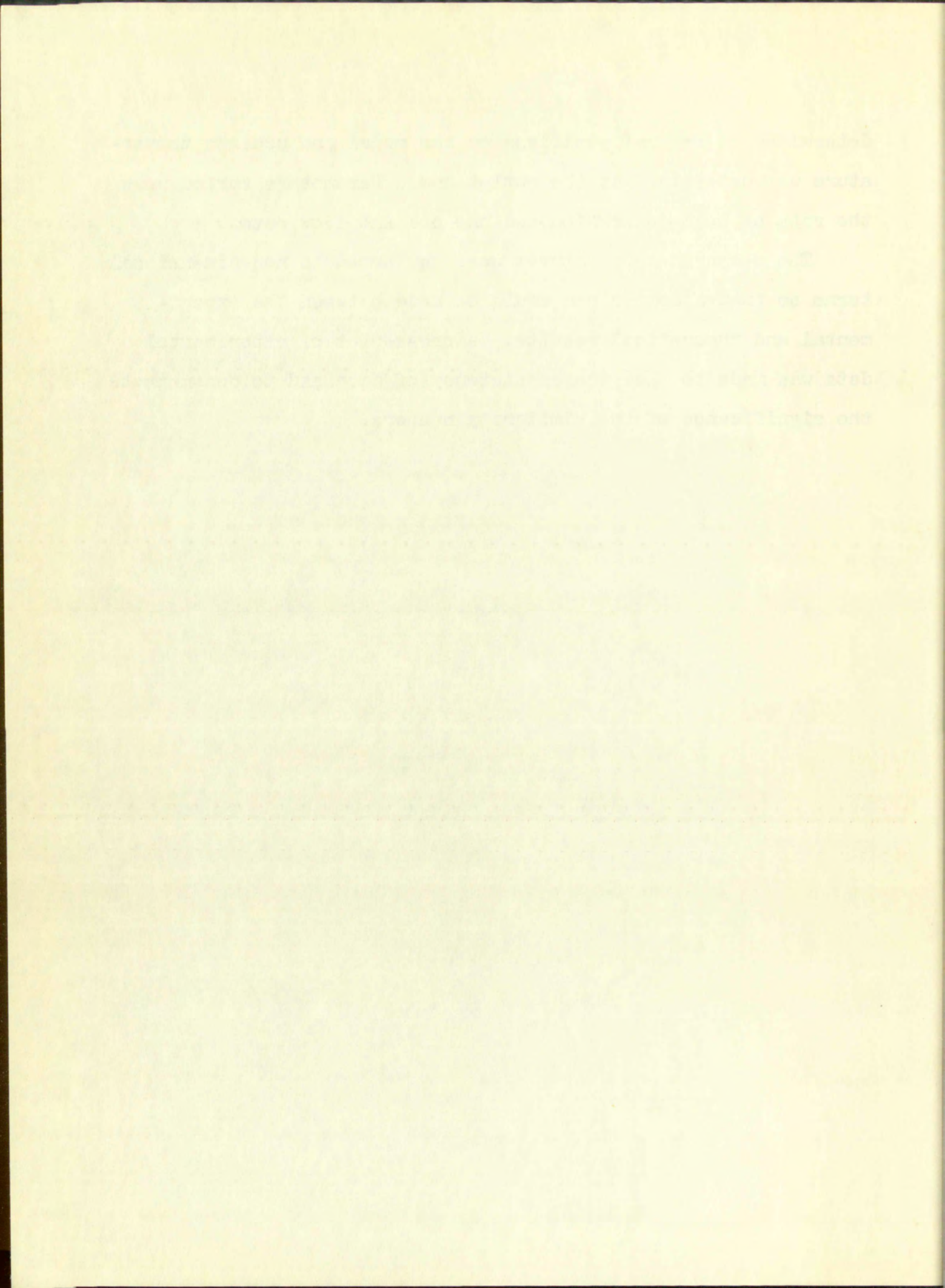
The problem of this investigation is the transient response of the full reactor and the fuel temperature of a nuclear reactor as a result of a power excursion. The problem is attacked both theoretically and experimentally.

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The experimental investigations result in measured temperature profiles of a typical model of a reactor fuel and coolant channels. The time variation of the fuel temperature was

determined at several positions on the model and coolant temperature was determined at the outlet end. Parameters varied were the rate of heat generation and the coolant flow rate.

The measured temperatures were converted to non-dimensional terms so that a comparison could be made between the experimental and theoretical results. A cross-plot of experimental data was made to show the consistency of data and to demonstrate the significance of the similarity numbers.



CHAPTER 2

LITERATURE REVIEW

Analysis of the complete nuclear reactor response to transient changes in reactivity has been presented by several authors in the unclassified literature. Notable among these is Schultz(1)⁺ who lumps spacial effects and makes other simplifying assumptions that permit him to express heat transfer phenomenon in the form of "transfer functions." These are combined with the transfer functions of other reactor processes associated with the transient to give a prediction of the response of the entire nuclear system. This technique omits details of the heat transfer transient to permit solution of the larger problem. A more complex treatment is presented by Howard(2) who considers in some detail the average cell in a heterogeneous reactor. Although the heat transfer process is given considerable attention, spacial variations are averaged to permit more emphasis to be placed on the "neutronics" of the problem.

Exclusive attention is given to the thermodynamic aspects of reactor transients in the paper by Brown(3) who emphasized that the transient analysis must be based on the generalized

⁺Numbers in parentheses refer to the LIST OF REFERENCES in the Appendix.

The first part of the report describes the general situation in the country and the progress of the work done during the year. It is followed by a detailed account of the various projects which have been carried out, and a summary of the results obtained. The report concludes with some observations on the work done during the year and a statement of the plans for the future.

The work done during the year has been of a very high standard, and it is a pleasure to report that the progress made has been very satisfactory. The various projects which have been carried out have all been completed, and the results obtained have been very good. It is hoped that the work done during the year will be of great value to the country, and that it will lead to further progress in the future.

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laws of thermodynamics rather than on a succession of steady state equations. An approximate method for predicting the rise of reactor temperatures following start-up is presented in an article by Bonilla(4). His solution is achieved by numerical solution of finite steps in space and time.

Hellman, Habetler, and Barbrov(5) have set up the thermal transient problem for the whole core of a liquid metal-cooled reactor in the form of a thermal network for numerical solution. Heat generation and transmission are considered as functions of time and two space variables. Temperatures and thermal properties are space and time averaged for each space and time node. This leads to more than 110 finite difference equations and solution was accomplished on a high-speed digital computer. The application of numerical methods to transient problems involving a perturbation in initial coolant temperature or direction of coolant flow has been presented by Dusenberre(6).

Paynter and Takahashi(7) have published a method for computing the response of heat exchangers from an analogy between their equations and those from statistics. The response of a fluid flowing through an insulated pipe subject to a step increase in the inlet temperature has been published by Rizika(8,9) for both compressible and incompressible flow.

By far the most pertinent work is that of Clark, Arpaci, and Treadwell(10,11) which appeared while this investigation was in progress. These authors had derived differential

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equations by a similar method and with the same assumptions as those presented in this thesis. Their solutions were also obtained by use of the Laplace transform and are similar. Improvement in the form of the solutions results from the use of non-dimensional equations here and from the use of alternate mathematical methods of inversion. The experimental verification exhibited with the papers by Clark, et al, was contributed by Treadwell(12) and Stuart(13) in their thesis work at Massachusetts Institute of Technology. Their experiments were performed on equipment nearly identical to that used in this investigation and included the measurement of fluid and metal temperatures at two locations for temperature changes on the order of 50°F. Their coolant fluid was water whereas air was used here.

Numerical solutions to the non-dimensional differential equations were calculated by Bankston(14) in connection with this project.

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CHAPTER 3

THEORETICAL INVESTIGATION

A physical system typical of this problem may be described as a solid cylindrical metal fuel rod within a concentric circular channel through the moderating material. The annular space is filled with pressurized gas which flows in the positive x direction. The transient increase in power occurs at time $t = 0$. A cross-section of a differential element is shown in Figure 1.

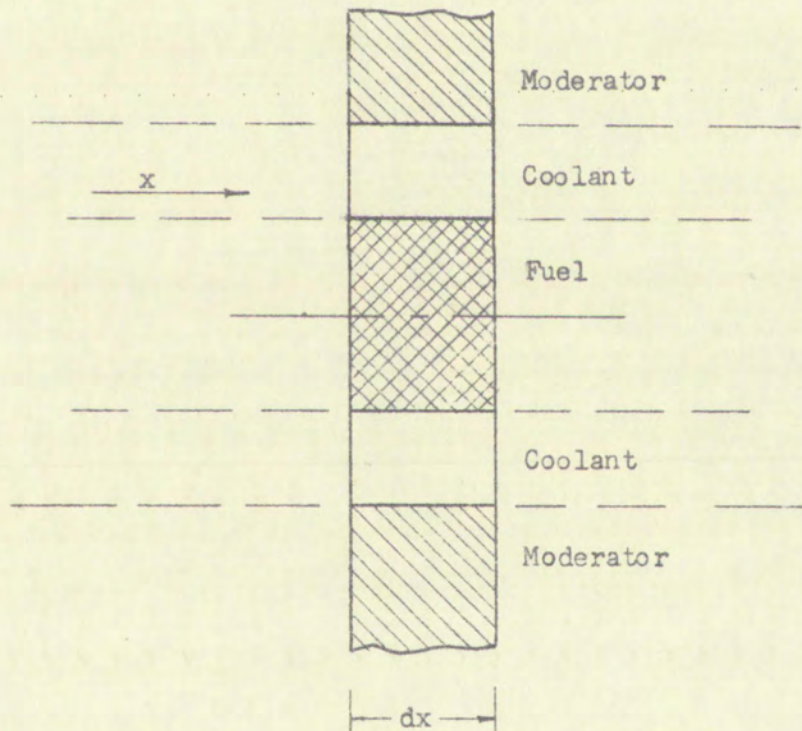


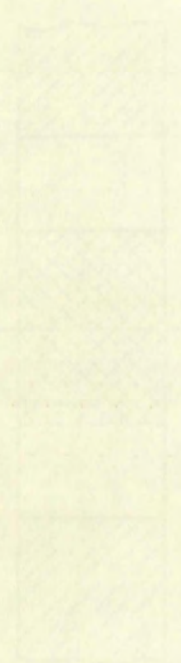
Figure 1 Differential Cross-section of a Fuel Element
and Its Cooling Channel

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The theoretical problem is simplified by the following assumptions:

1. The interface between moderator and coolant is adiabatic.
2. There is a stepwise increase in power with respect to time.
3. There are no radial temperature gradients in either the metal or the fluid, but there is convection heat transfer from the fuel to the gas.
4. There is no axial heat conduction.
5. All metal and fluid properties are constant.
6. The heat transfer coefficient $h \neq f(x,t)$.

DERIVATION OF DIFFERENTIAL EQUATIONS

The differential section in Figure 1 may be regarded as two parts: the fluid part and the metal part. Each of these yields an energy balance equation. For the metal part

$$(U_g + \Delta U_g) dx = h p (m - f) dx + c_m \gamma_m A_m \left(\frac{\partial m}{\partial t} \right) dx \quad 1$$

and for the fluid part

$$h p (m - f) = w c_f df \quad 2$$

where U_g = steady power, BTU/hr

ΔU_g = step increase in power, BTU/hr

h = heat transfer coefficient, BTU/f²hr^oF

p = perimeter of fuel element, f

1. The first part of the paper is devoted to the study of the properties of the function $f(x)$ defined by the equation

$$f(x) = \int_0^x f(t) dt + x^2$$

It is shown that the function $f(x)$ is a polynomial of degree 2 and that its coefficients are determined by the initial conditions

$$f(0) = 0, \quad f'(0) = 1$$

It is also shown that the function $f(x)$ satisfies the differential equation

$$f''(x) = 2x$$

The general solution of this equation is

$$f(x) = x^2 + C_1 x + C_2$$

where C_1 and C_2 are constants. The initial conditions determine the constants C_1 and C_2 uniquely.

It is also shown that the function $f(x)$ is a solution of the differential equation

$$f''(x) = 2x$$

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$$f''(x) = 2x$$

m = metal temperature, °F

f = fluid temperature, °F

c = specific heat (at constant pressure for the fluid),
BTU/lb°F

δ = specific weight, lb/f³

A = cross-sectional area in radial plane, f²

w = fluid flow rate, lb/hr

and the subscripts m and f refer to the metal and fluid parts.

Since $f = f(x, t)$, then

$$df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt.$$

Also $w = \delta_f A_f V$ where V is the fluid velocity.

Equations 1 and 2 may be rearranged to form

$$\begin{aligned} (m - f) &= \frac{U_g + \Delta U_g}{hp} - \frac{c_m \delta_m A_m}{hp} \left(\frac{\partial m}{\partial t} \right) \\ &= \frac{U_g + \Delta U_g}{hp} - \frac{c_m \delta_m A_m}{hp} \left(\frac{\partial m}{\partial t} \right) \end{aligned} \quad 3$$

and

$$(m - f) = \frac{w c_f}{hp} \left(\frac{\partial f}{\partial x} \right) + \frac{c_f \delta_f A_f}{hp} \left(\frac{\partial f}{\partial t} \right). \quad 4$$

The steady and transient parts of equations 3 and 4 may be separated by the substitution

$$m = m' + m'' \text{ and } f = f' + f''$$

where the single prime denotes the steady state temperature for $t < 0$ when $\frac{\partial m'}{\partial t} = \frac{\partial f'}{\partial t} = 0$, and the double prime denotes the

The first part of the proof is to show that the function $f(x)$ is continuous at x_0 . Let $\epsilon > 0$ be given. We need to find a $\delta > 0$ such that if $|x - x_0| < \delta$, then $|f(x) - f(x_0)| < \epsilon$.

Since $f(x) = \frac{1}{x}$, we have $|f(x) - f(x_0)| = \left| \frac{1}{x} - \frac{1}{x_0} \right| = \frac{|x - x_0|}{|x x_0|}$.

We want to bound this expression. Note that $|x x_0| = |x| |x_0|$. If $|x - x_0| < \delta$, then $|x| > |x_0| - \delta$. We choose $\delta < |x_0|$, so $|x| > |x_0|/2$.

Therefore, $|f(x) - f(x_0)| < \frac{\delta}{|x_0|^2/2} = \frac{2\delta}{|x_0|^2}$. We want this to be less than ϵ , so we choose $\delta < \frac{\epsilon |x_0|^2}{2}$.

Also, we can show that $f(x)$ is not continuous at $x = 0$. Suppose $f(x)$ were continuous at $x = 0$. Then for $\epsilon = 1$, there would be a $\delta > 0$ such that $|f(x) - f(0)| < 1$ whenever $|x - 0| < \delta$. But $f(0)$ is not defined, so this is impossible.

The second part of the proof is to show that $f(x)$ is not continuous at $x = \infty$. Suppose $f(x)$ were continuous at $x = \infty$. Then for $\epsilon = 1$, there would be a $M > 0$ such that $|f(x) - f(\infty)| < 1$ whenever $|x - \infty| < M$. But $f(x) = \frac{1}{x}$ approaches 0 as $x \rightarrow \infty$, so $f(\infty) = 0$. This would mean $|f(x) - 0| < 1$ for $|x - \infty| < M$, which is not true.

transient part. Then

$$m' - f' + m'' - f'' = \frac{U_g}{hp} + \frac{\Delta U_g}{hp} - \frac{c_m \partial_m^A m}{hp} \left(\frac{\partial m''}{\partial t} \right) \quad 5$$

and

$$m' - f' + m'' - f'' = \frac{w c_f}{hp} \left(\frac{\partial f'}{\partial x} + \frac{\partial f''}{\partial x} \right) + \frac{c_f \partial_f^A f}{hp} \left(\frac{\partial f''}{\partial t} \right) \quad 6$$

But when $t < 0$, $m' - f' = \frac{U_g}{hp}$ 7

$$m' - f' = w c_f \left(\frac{\partial f'}{\partial x} \right). \quad 8$$

By combining equations 5, 6, 7, and 8, the differential equations for $t \geq 0$ become

$$m'' - f'' = \frac{\Delta U_g}{hp} - \frac{c_m \partial_m^A m}{hp} \left(\frac{\partial m''}{\partial t} \right) \quad 9$$

$$m'' - f'' = \frac{w c_f}{hp} \left(\frac{\partial f''}{\partial x} \right) + \frac{c_f \partial_f^A f}{hp} \left(\frac{\partial f''}{\partial t} \right) \quad 10$$

This illustrates that the transient parts of the temperatures are independent and may be superimposed upon the steady state temperature profiles. Henceforth, the double primes will be omitted but it will be recalled that temperatures refer only to the transient part.

DIMENSIONAL ANALYSIS

A further simplification results if equations 9 and 10 are converted to non-dimensional form. The number of similarity numbers is determined by the Buckingham Pi theorem. In this

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problem the number of variables $n = 9$: (m) , (f) , (x) , (t) , $(w c_f)$, $(c_f \delta_f A_f)$, $(c_m \delta_m A_m)$, (ΔU_g) , and (hp) . The number of primary independent units $p = 4$: (mass), (length), (time), and (temperature). Then the number of similarity numbers to be expected is $n - p = 5$. Since m and f of the original nine variables are dependent, then there should be 2 dependent and 3 independent similarity numbers.

An inspection of the differential equations 9 and 10 indicates that two of the independent similarity numbers are

$$X = \frac{xhp}{wc_f} \quad \text{and} \quad T = \frac{thp}{c_f \delta_f A_f}$$

where X and T are non-dimensional lengths and time. The third independent similarity number appears later in the derivation. The two dependent similarity numbers are expressed initially as $m = K_m M$ and $f = K_f F$ where M and F are non-dimensional temperatures. Then

$$\left(\frac{\partial m}{\partial t}\right) = K_m \left(\frac{\partial M}{\partial T}\right) \left(\frac{\partial T}{\partial t}\right) = K_m \left(\frac{hp}{c_f \delta_f A_f}\right) \left(\frac{\partial M}{\partial T}\right)$$

$$\left(\frac{\partial f}{\partial x}\right) = K_f \left(\frac{\partial F}{\partial X}\right) \left(\frac{\partial X}{\partial x}\right) = K_f \left(\frac{hp}{wc_f}\right) \left(\frac{\partial F}{\partial X}\right)$$

$$\left(\frac{\partial f}{\partial t}\right) = K_f \left(\frac{hp}{c_f \delta_f A_f}\right) \left(\frac{\partial F}{\partial T}\right)$$

Substituting the above into 9 and 10 results in

$$K_m M - K_f F = \frac{\Delta U_g}{hp} - K_m \left(\frac{c_m \delta_m A_m}{c_f \delta_f A_f}\right) \left(\frac{\partial M}{\partial T}\right)$$

The following table shows the results of the experiments conducted on the effect of temperature on the rate of reaction between hydrogen peroxide and potassium iodide. The rate of reaction was measured by the volume of oxygen gas evolved in a given time.

Temperature (°C)	Rate of Reaction (ml O ₂ / min)
10	1.5
20	3.0
30	6.0
40	12.0
50	24.0

From the above table, it is clear that the rate of reaction increases with an increase in temperature. This is because the molecules of the reactants possess more kinetic energy at higher temperatures, which enables them to overcome the activation energy barrier more easily.

$$K_m M - K_f F = K_f \left(\frac{\partial F}{\partial T} \right) + K_f \left(\frac{\partial F}{\partial X} \right)$$

It is now apparent that the third similarity number ought to be

$$a = \frac{c_m \delta_m \Lambda_m}{c_f \delta_f \Lambda_f} \text{ and for the simplest form } K_m = K_f = \frac{\Delta U_g}{hp}$$

Now the differential equations to be solved are:

$$M - F = 1 - a \frac{\partial M}{\partial T} \quad 11$$

$$M - F = \frac{\partial F}{\partial X} + \frac{\partial F}{\partial T} \quad 12$$

where the similarity numbers are defined as $M \equiv \frac{mhp}{\Delta U_g}$, $F \equiv \frac{fhp}{\Delta U_g}$,

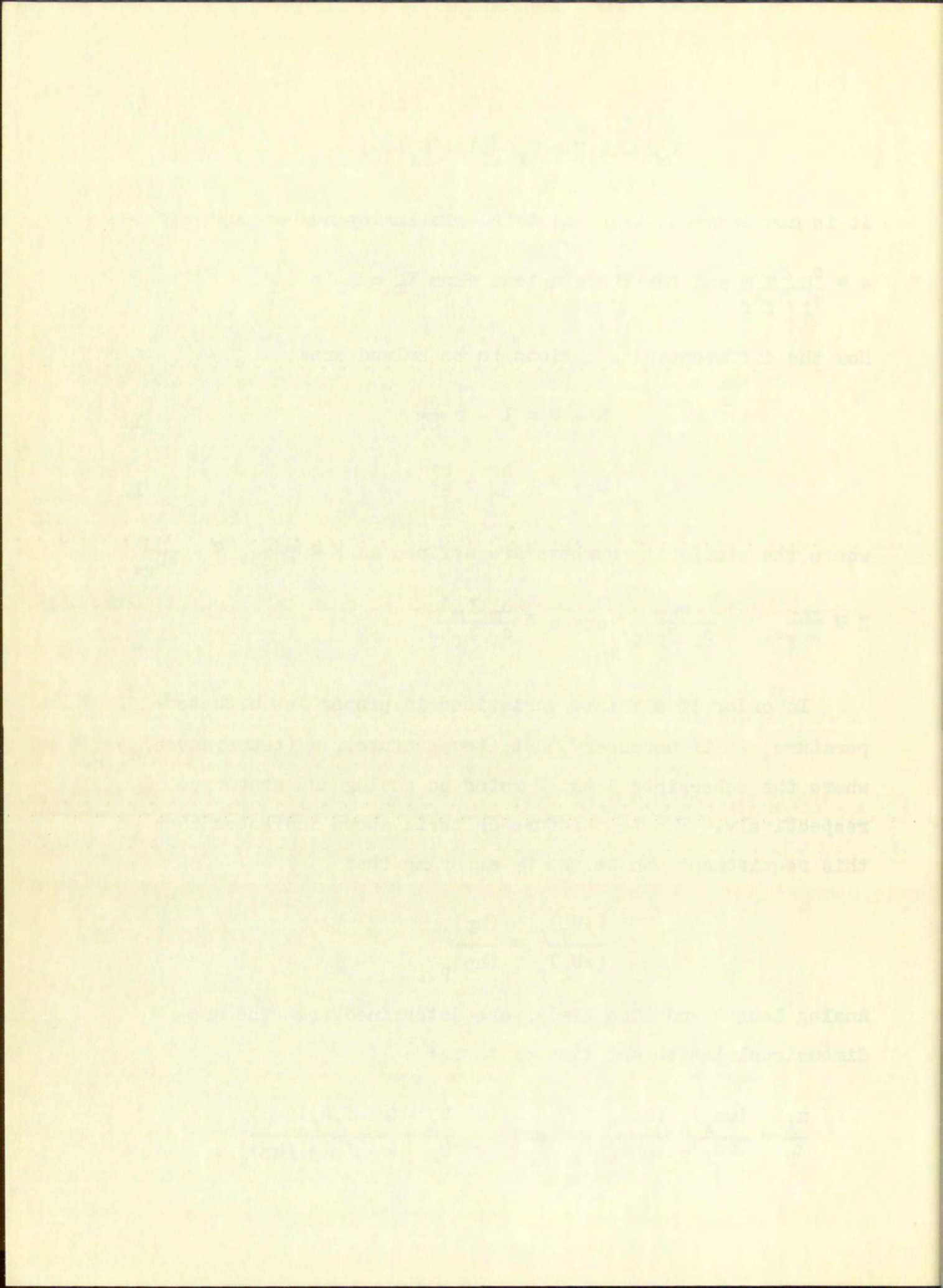
$$X \equiv \frac{xhp}{wc_f}, \quad T \equiv \frac{thp}{c_f \delta_f \Lambda_f}, \quad \text{and } a \equiv \frac{c_m \delta_m \Lambda_m}{c_f \delta_f \Lambda_f}.$$

In order to simulate variations in properties with temperature, it is necessary that $(\text{temperature})_A = (\text{temperature})_P$ where the subscripts A and P refer to analog and prototype respectively. The temperature criteria above indicates that this requirement can be met by assuring that

$$\frac{(\Delta U_g)_A}{(\Delta U_g)_P} = \frac{(hp)_A}{(hp)_P}.$$

Analog length and time scales are determined from the non-dimensional length and time criteria:

$$\frac{x_A}{x_P} = \frac{(wc_f)_A}{(wc_f)_P} \frac{(hp)_P}{(hp)_A} \quad \text{and} \quad \frac{t_A}{t_P} = \frac{(c_f \delta_f \Lambda_f)_P}{(c_f \delta_f \Lambda_f)_A} \frac{(hp)_P}{(hp)_A}.$$



ANALYTICAL SOLUTION

Equations 11 and 12 may be solved explicitly for M and F to obtain

$$\frac{\partial^2 F}{\partial T^2} + \frac{\partial^2 F}{\partial XT} + (1 + 1/a)\frac{\partial F}{\partial T} + (1/a)\frac{\partial F}{\partial X} - (1/a) = 0 \quad 13$$

and

$$\frac{\partial^2 M}{\partial T^2} + \frac{\partial^2 M}{\partial XT} + (1 + 1/a)\frac{\partial M}{\partial T} + (1/a)\frac{\partial M}{\partial X} - (1/a) = 0 \quad 14$$

Equations 13 and 14 may be operated on by the Laplace transform method in which the needed transformations are found in the references(15,16,17). The following transforms will be used:

$$L[1 - e^{-T/a}] = \frac{a}{p(p + 1/a)}$$

$$L[g(X,T)] = \int_0^\infty e^{-pt}[g(X,T)dT] = \bar{g}(X,p)$$

$$L\left[\frac{\partial^2 g}{\partial T^2}\right] = p^2\bar{g} - p[g(X,+0)] - \frac{\partial g}{\partial T}(X,+0)$$

$$L\left[\frac{\partial^2 g}{\partial X \partial T}\right] = \frac{\partial}{\partial X}[p\bar{g} - g(X,+0)]$$

$$L\left[\frac{\partial g}{\partial T}\right] = p\bar{g} - g(X,+0)$$

$$L\left[\frac{\partial g}{\partial X}\right] = \frac{\partial \bar{g}}{\partial X}$$

$$L[1/a] = 1/ap$$

The initial conditions imposed on this problem are:

a. $F(X,+0) = 0$

c. $M(X,+0) = 0$

b. $\frac{\partial F}{\partial T}(X,+0) = 0$

d. $\frac{\partial M}{\partial T}(X,+0) = 1/a$

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Hence the transformed equation 13 is

$$p^2 \bar{F} + \frac{\partial}{\partial X}(p \bar{F}) + (1 + 1/a)p \bar{F} + (1/a) \frac{\partial \bar{F}}{\partial X} - 1/ap = 0$$

or

$$\frac{\partial \bar{F}}{\partial X} + \frac{p^2 + p + p/a}{p + 1/a} \bar{F} = \frac{1/a}{p(p + 1/a)} \quad 15$$

and the transformed equation 14 is

$$p^2 \bar{M} - 1/a + \frac{\partial}{\partial X}(p \bar{M}) + (1 + 1/a)p \bar{M} + (1/a) \frac{\partial \bar{M}}{\partial X} - 1/ap = 0$$

or

$$\frac{\partial \bar{M}}{\partial X} + \frac{p^2 + p + p/a}{p + 1/a} \bar{M} = \frac{1/a(p + 1)}{p(p + 1/a)}. \quad 16$$

Equations 15 and 16 may be readily solved as ordinary differential equations. The boundary condition, $F(T,0) = 0$, $\bar{F}(p,0) = 0$, provides for the transformed solution of equation 15:

$$\bar{F}(X,p) = \frac{1/a}{p^2(p + 1 + 1/a)} \left[1 - \exp - \frac{p(p + 1 + 1/a)X}{(p + 1/a)} \right]. \quad 17$$

A suitable boundary condition for equation 16 may be obtained from equation 11 where $F(0,T) = 0$ so that

$$\frac{\partial M}{\partial T}(0,T) + (1/a) M(0,T) = 1/a.$$

Operating on this by the Laplace transform yields

$$p \bar{M}(0,p) - M(0,+0) + (1/a) \bar{M}(0,p) = (1/ap) \text{ or } \bar{M}(0,p) = \frac{1/a}{p(p + 1/a)}$$

since $M(X,+0) = 0$. Then the transformed solution for equation 16 is

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$$\bar{M}(X,p) = \frac{1/a(p+1)}{p^2(p+1+1/a)} - \left[\frac{1/a^2}{p^2(p+1+1/a)(p+1/a)} \right] \exp - \frac{p(p+1+1/a)X}{p+1/a} X. \quad 18$$

The inversion of equations 17 and 18 is simplified by expanding the first fractions in each into partial fractions and by performing the division in the exponential. It is convenient to let $b \equiv 1 + 1/a$. Then

$$\bar{F}(X,p) = -\frac{1}{ab^2p} + \frac{1}{abp^2} + \frac{1}{ab^2(p+b)} - \frac{1}{ap^2(p+b)} \left[\exp - pX - X + \frac{X/a}{p+1/a} \right] \quad 19$$

and

$$\bar{M}(X,p) = \frac{1}{a^2b^2p} + \frac{1}{abp^2} - \frac{1}{a^2b^2(p+b)} - \frac{1/a^2}{p^2(p+b)(p+1/a)} \left[\exp - pX - X + \frac{X/a}{p+1/a} \right] \quad 20$$

Difficulty arises in obtaining the inverse solution because of the term $\exp \frac{X/a}{p+1/a}$. However, this may be expanded into the power series $1 + \frac{(X/a)(p+1/a)^{-1}}{1!} + \frac{(X/a)^2(p+1/a)^{-2}}{2!} + \dots + \frac{(X/a)^n(p+1/a)^{-n}}{n!}$ $n = 1, 2, 3, \dots$

An alternate approach to this difficulty is to arrange terms involving p such that the inverse is one of the Bessel functions(10,11). However, difficulty arises later when performing

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right)$$

The inverse of equation (1) and (2) is obtained by expanding the first two terms in each into partial fractions and by performing the division in the exponential. It is convenient to let $\alpha = 1/\sqrt{2}$. Then

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \exp\left(-\frac{y^2}{2}\right)$$

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \exp\left(-\frac{y^2}{2}\right)$$

and

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \exp\left(-\frac{y^2}{2}\right)$$

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \exp\left(-\frac{y^2}{2}\right)$$

Equation (1) is obtained by substituting the first two terms of the series $\exp\left(-\frac{x^2}{2}\right)$ and $\exp\left(-\frac{y^2}{2}\right)$ into the

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \exp\left(-\frac{y^2}{2}\right)$$

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2 + y^2}{2}\right) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \exp\left(-\frac{y^2}{2}\right)$$

An alternate approach to this difficulty is to express the

involving α such that the division in the exponential is

the integration in the convolution method. Some assistance might be provided here by referring to the work by Wheelon and Robacker(18).

The terms in equation 19 and 20 must now be properly grouped.

$$\begin{aligned} \bar{F}(X,p) = & -\frac{1}{ab^2p} + \frac{1}{abp^2} + \frac{1}{ab^2(p+b)} \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \right] \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \sum_1^{\infty} \frac{(X/a)^n (p+1/a)^{-n}}{n!} \right] \end{aligned} \quad 21$$

$$\begin{aligned} \bar{M}(X,p) = & \frac{1}{a^2b^2p} + \frac{1}{abp^2} - \frac{1}{a^2b^2(p+b)} \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \left(\frac{1}{p+1/a} \right) \right] \\ & - \frac{e^{-X}}{a} \left[\left(\frac{e^{-pX}}{p^2} \right) \left(\frac{1}{p+b} \right) \left(\frac{1}{p+1/a} \right) \sum_1^{\infty} \frac{(X/a)^n (p+1/a)^{-n}}{n!} \right] \end{aligned} \quad 22$$

The inverse functions required are as follows:

		<u>Restrictions</u>
$L^{-1}[(p+1/a)^{-n}] = \frac{T^{n-1}e^{-T/a}}{(n-1)!}$		$T > 0$
$L^{-1}[1/p] = 1$		$T > 0$
$L^{-1}[1/p^2] = T$		$T > 0$
$L^{-1}\left[\frac{1}{p+b}\right] = e^{-bT}$		$T > 0$
$L^{-1}\left[\frac{1}{p+1/a}\right] = e^{-T/a}$		$T > 0$
$L^{-1}\left[\frac{e^{-pX}}{p^2}\right] = 0$ $= T - X$		$0 < T < X$ $T > X$

The restrictions on the term $(T - X)$ implies that the inverse solutions must be handled separately for each of two time intervals: $0 \leq T \leq X$ and $T > X$. Referring to the definitions, $T > X$ is equivalent to $t > x/V$ where V is the fluid velocity. This has the physical significance that until $t = x/V$ no fresh fluid has reached point x and the temperatures are independent of x . After $t = x/V$, fluid which was not in the channel at $t = 0$ influences temperatures at x and the temperatures are dependent on x .

The solutions of 19 and 20 for the first time interval are

$$F(X, T) = T/ab + 1/ab^2[e^{-bT} - 1] \quad 23$$

and

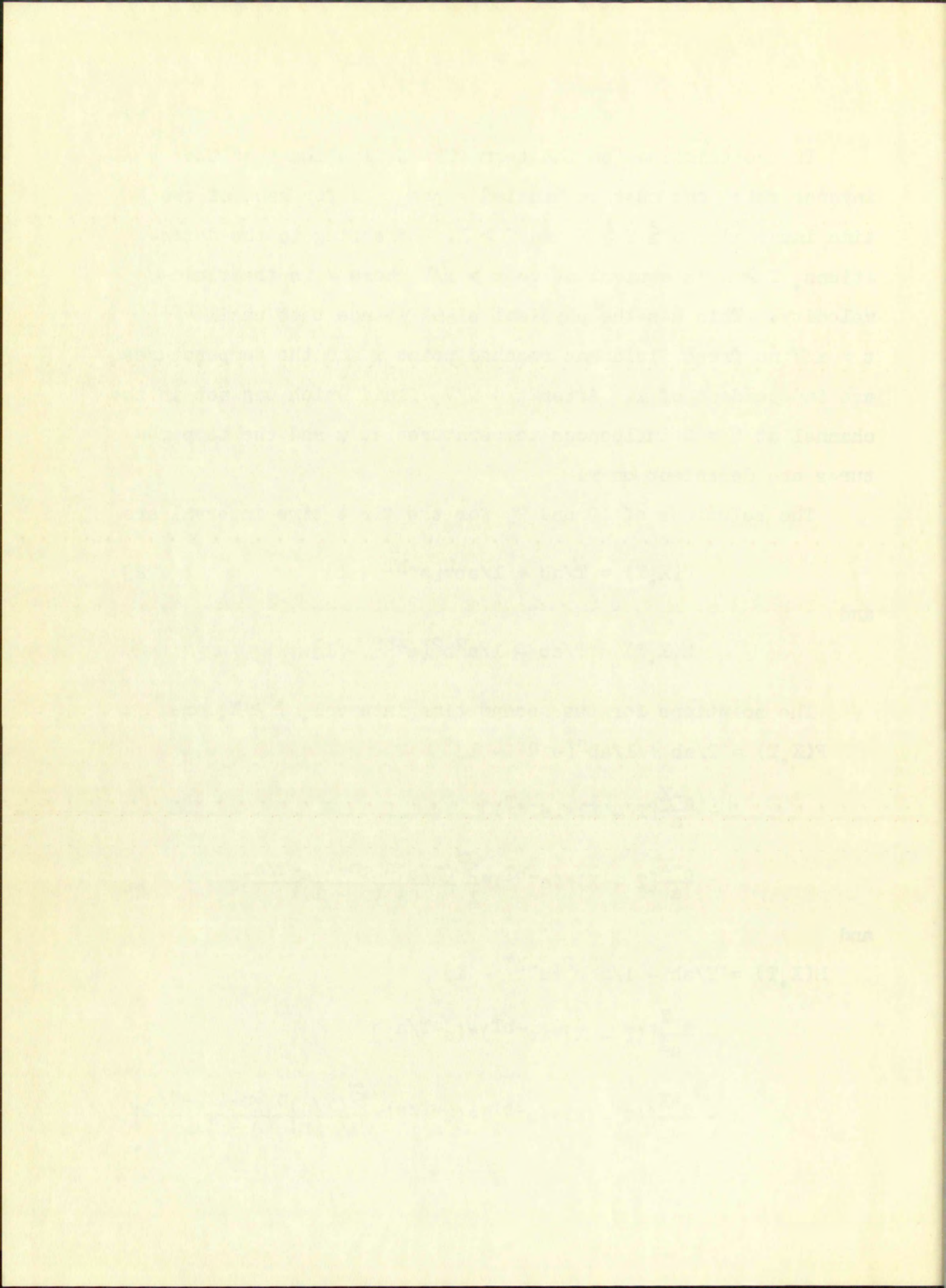
$$M(X, T) = T/ab - 1/a^2b^2[e^{-bT} - 1]. \quad 24$$

The solutions for the second time interval, $T > X$, are

$$\begin{aligned} F(X, T) &= T/ab + 1/ab^2[e^{-bT} - 1] \\ &- \frac{e^{-X}}{a}[T - X](e^{-bT}) \\ &- \frac{e^{-X}}{a}[T - X](e^{-bT}) * \sum_1^{\infty} \frac{(X/a)^n T^{n-1} e^{-T/a}}{n! (n-1)!} \end{aligned} \quad 25$$

and

$$\begin{aligned} M(X, T) &= T/ab - 1/a^2b^2[e^{-bT} - 1] \\ &- \frac{e^{-X}}{a^2}[(T - X)(e^{-bT})(e^{-T/a})] \\ &- \frac{e^{-X}}{a^2}[(T - X)(e^{-bT})(e^{-T/a}) * \sum_1^{\infty} \frac{(X/a)^n T^{n-1} e^{-T/a}}{n! (n-1)!}] \end{aligned} \quad 26$$



where the asterisks indicate that the terms so connected are to be treated by the method of convolution. This is a technique for the inverse transformation for the product of functions and is discussed by Churchill(16), pages 36-40 and by Pipes(17), pages 525-526. The following equations illustrate the general convolution technique:

$$\bar{g}_1(X,p) \bar{g}_2(X,p) = g_1(X,T) * g_2(X,T)$$

$$\begin{aligned} g_1(X,T) * g_2(X,T) &= \int_0^T g_1(X,T-z) g_2(X,z) dz \\ &= \int_0^X g_1(X,T-z) g_2(X,z) dz \\ &\quad + \int_X^T g_1(X,T-z) g_2(X,z) dz \end{aligned}$$

In this problem, the inverse transformation of $\frac{e^{-pX}}{p^2}$ is involved in all the convolution operations. When $T < X$ the inverse is zero and therefore the integral from 0 to X is zero. Equations 25 and 26 are restricted in application to $T > X$ and the convolution integral has the limits X to T.

The convolutions required for equations 25 and 26 are now performed.

In equation 25

$$\begin{aligned} (T - X) * (e^{-bT}) &= \int_X^T (T - z - X) e^{-bz} dz \\ &= (T - X) \int_X^T e^{-bz} dz - \int_X^T z e^{-bz} dz \end{aligned}$$

where the order of integration is the same as in (15) and the limits are indicated by the subscripts. This is a consequence of the fact that the inverse transformation for the product of functions and is obtained by (16) and (17) respectively. The following equations illustrate the general convolution theorem:

$$\begin{aligned}
 \mathcal{L}\{f(x)g(x)\} &= \mathcal{L}\{f(x)\} \mathcal{L}\{g(x)\} \\
 \mathcal{L}\{f(x)g(x)\} &= \int_0^T f(x)g(x) e^{-sx} dx \\
 &= \int_0^T f(x)g(x) e^{-sx} dx \\
 &= \int_0^T f(x)g(x) e^{-sx} dx
 \end{aligned}$$

In this problem, the inverse transformation of $\frac{e^{-sx}}{p}$ is involved in all the convolution operations. When $T < X$ the inverse is zero and therefore the integral from 0 to X is zero. Equations (18) and (19) are verified in application to $T > X$ and the convolution integral has the limits X to T . The convolution is verified for equations (18) and (19) and is performed.

In equation (18)

$$\mathcal{L}\{f(x)g(x)\} = \int_0^T f(x)g(x) e^{-sx} dx$$

$$\mathcal{L}\{f(x)g(x)\} = \int_0^T f(x)g(x) e^{-sx} dx$$

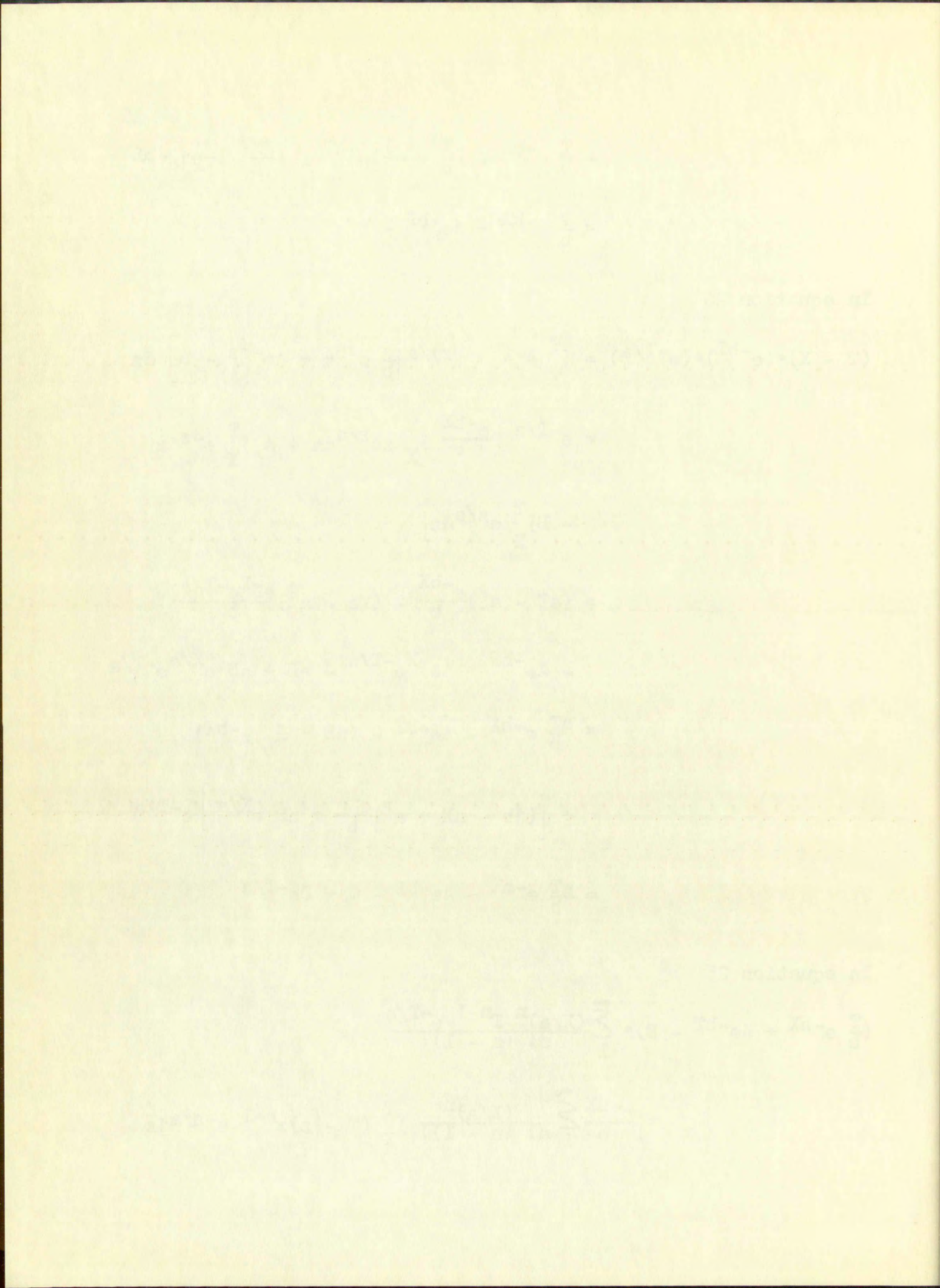
$$\begin{aligned}
&= \frac{T}{b} e^{-bX} + \left[\frac{X}{b} + \frac{1}{b^2} \right] e^{-bT} - \left[\frac{2X}{b} + \frac{1}{b^2} \right] e^{-bX} \\
&= \frac{T}{b} e^{-bX} + Ae^{-bT} - B
\end{aligned}$$

In equation 26

$$\begin{aligned}
(T - X)(e^{-bT})(e^{-T/a}) &= \int_X^T e^{-(T-z)/a} \left[\frac{z}{b} e^{-bX} + Ae^{-bz} - B \right] dz \\
&= e^{-T/a} \left[\frac{e^{-bX}}{b} \int_X^T ze^{z/a} dz + A \int_X^T e^{-z} dz \right. \\
&\quad \left. - B \int_X^T e^{z/a} dz \right] \\
&= (aT - a^2) \frac{e^{-bX}}{b} - (Xa - a^2) \frac{e^{-X} e^{-T/a}}{b} \\
&\quad - Ae^{-bT} + Ae^{-X} e^{-T/a} - aB + aB e^{X/a} e^{-T/a} \\
&= \frac{aT}{b} e^{-bX} - Ae^{-bT} - (aB + \frac{a^2}{b} e^{-bX}) \\
&\quad + \left[(Ab - aX + a^2) \frac{e^{-X}}{b} + aB e^{X/a} \right] e^{-T/a} \\
&= \frac{aT}{b} e^{-bX} - Ae^{-bT} - C + De^{-T/a}
\end{aligned}$$

In equation 25

$$\begin{aligned}
\left(\frac{T}{b} e^{-bX} + Ae^{-bT} - B \right) * \sum_1^{\infty} \frac{(X/a)^n T^{n-1} e^{-T/a}}{n! (n-1)!} &= \\
&= \frac{e^{-bX}}{b} \sum_1^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T (T-z) z^{n-1} e^{-z/a} dz
\end{aligned}$$



$$\begin{aligned}
& + A \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T e^{-b(T-z)} z^{n-1} e^{-z/a} dz \\
& - B \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T z^{n-1} e^{-z/a} dz
\end{aligned}$$

Without evaluating integrals, this is of the form

$$= \frac{e^{-bX}}{b} \Sigma_1 + A \Sigma_2 - B \Sigma_3$$

where Σ_1 , Σ_2 and Σ_3 involve combinations of series.

In equation 26

$$\begin{aligned}
& \left(\frac{aT}{b} e^{-bX} - Ae^{-bT} - C + De^{-T/a} \right) * \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \frac{T^{n-1} e^{-T/a}}{n! (n-1)!} = \\
& = \frac{a}{b} e^{-bX} \Sigma_1 - A \Sigma_2 - C \Sigma_3 \\
& + D \sum_{n=1}^{\infty} \frac{(X/a)^n}{n! (n-1)!} \int_X^T e^{-(T-z)/a} z^{n-1} e^{-z/a} dz \\
& = \frac{a}{b} e^{-bX} \Sigma_1 - A \Sigma_2 - C \Sigma_3 + D \Sigma_4
\end{aligned}$$

The following solutions for the second time interval are obtained by substituting the results of the convolutions in equations 25 and 26

$$\begin{aligned}
F(X, T) &= T/ab + 1/ab^2 (e^{-bT} - 1) \\
&- \frac{e^{-X}}{a} \left[(T + \Sigma_1) \frac{e^{-bX}}{b} + A(e^{-bT} + \Sigma_2) - B(1 + \Sigma_3) \right]
\end{aligned}$$

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The following relations for the
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equations

... ..

$$F(X,T) = T/ab + 1/ab^2(e^{-bT} - 1) - \frac{e^{-X}}{ab} \left[(T + \Sigma_1)e^{-bX} \right. \\ \left. + (X + 1/b)(e^{-bT} + \Sigma_2) - (2X + 1/b)(1 + \Sigma_3) \right] \quad 27$$

and

$$M(X,T) = T/ab - 1/a^2b^2(e^{-bT} - 1) \\ - \frac{e^{-X}}{a^2} \left[(T + \Sigma_1)\frac{a}{b} e^{-bX} - A(e^{-bT} + \Sigma_2) - C(1 + \Sigma_3) \right. \\ \left. + D(e^{-T/a} + \Sigma_4) \right] \\ = T/ab - 1/a^2b^2(e^{-bT} - 1) - \frac{e^{-X}}{ab} \left[(T + \Sigma_1)e^{-bX} \right. \\ - (X/a + 1/ab)(e^{-bT} + \Sigma_2) \\ - e^{-bX} [(2X + 1/b) + a](1 + \Sigma_3) \\ \left. + [e^{-X}(X/a + 1/ab - X + a) + e^{-bX}(2X + 1/b)](e^{-T/a} + \Sigma_4) \right]. \quad 28$$

All of the above integrals are of the form

$$\int z^n e^{az} dz = e^{az} \left[\frac{z^n}{a} - \frac{n z^{n-1}}{a^2} + \frac{n(n-1)}{a^3} z^{n-2} - \dots \right. \\ \left. + (-1)^{n-1} \frac{n! z}{a^n} + (-1)^n \frac{n!}{a^{n+1}} \right],$$

so that Σ_1 , Σ_2 , Σ_3 and Σ_4 involve combinations of series which would be difficult to evaluate without a digital computer. For that reason a numerical solution of these equations was not attempted.

$$M^{-1} \left[\frac{1}{s^2} + \frac{1}{s} \right] = \frac{1}{s^2} + \frac{1}{s} = \frac{1 + s}{s^2} = \frac{1}{s^2} + \frac{1}{s}$$

$$\left[(s^2 + 1)(s + 1) - (s^2 + 1) \right] \frac{1}{s^2} = \frac{1}{s^2}$$

and

$$M^{-1} \left[\frac{1}{s^2} + \frac{1}{s} \right] = \frac{1}{s^2} + \frac{1}{s} = \frac{1 + s}{s^2} = \frac{1}{s^2} + \frac{1}{s}$$

$$\left[(s^2 + 1)(s + 1) - (s^2 + 1) \right] \frac{1}{s^2} = \frac{1}{s^2}$$

$$M^{-1} \left[\frac{1}{s^2} + \frac{1}{s} \right] = \frac{1}{s^2} + \frac{1}{s} = \frac{1 + s}{s^2} = \frac{1}{s^2} + \frac{1}{s}$$

$$(s^2 + 1)(s + 1) - (s^2 + 1) = 1$$

$$M^{-1} \left[\frac{1}{s^2} + \frac{1}{s} \right] = \frac{1}{s^2} + \frac{1}{s} = \frac{1 + s}{s^2} = \frac{1}{s^2} + \frac{1}{s}$$

$$(s^2 + 1)(s + 1) - (s^2 + 1) = 1$$

All of the above integrals are of the form

$$\int_0^{\infty} e^{-st} f(t) dt = \frac{1}{s} \left[\frac{1}{s} + \frac{1}{s} \right] = \frac{2}{s^2}$$

$$\left[\frac{1}{s} + \frac{1}{s} \right] = \frac{2}{s}$$

so that I_1, I_2, I_3 and I_4 involve combinations of entries which
 would be difficult to evaluate without a digital computer. For
 that reason a numerical solution of these equations was not
 attempted.

It has not been possible to show that this solution converges to a steady state solution for $T \rightarrow \infty$ because of the appearance of T in the infinite series terms. However, a partial check of the final solution is provided by substituting $X = 0$ which gives back the boundary conditions $F(0, T) = 0$ and $M(0, T) = 1 - e^{-T/a}$.

The evaluation of the infinite series terms would amount to a tremendous undertaking without a high-speed digital computer. The series terms derived here can probably be manipulated to match the series terms given by Clark et al, (10, 11) which were computed for a range of parameters on the M.I.T.-I.B.M. digital computer.

In gas-cooled reactors, the value of "a" may be quite large for the low-power research models but becomes smaller and approaches one for power reactors. Equations 27 and 28 indicate that the effect of decreasing "a" is to increase the initial rate of temperature response.

NUMERICAL SOLUTION

The analytical solutions of equations 11 and 12 are so complex that a numerical solution is of more value when a digital computer is not available. Solution of the equations by a finite difference method was accomplished by Bankston(14) as a special problem for the Department of Mechanical Engineering, University of New Mexico. This investigation was in progress at that time and parameters were chosen to be

The first part of the paper is devoted to the study of the asymptotic behavior of the solutions of the system (1) as $t \rightarrow \infty$. It is shown that the solutions of (1) are bounded and tend to zero as $t \rightarrow \infty$ if the matrix A is stable. The second part of the paper is devoted to the study of the asymptotic behavior of the solutions of the system (1) as $t \rightarrow \infty$ if the matrix A is not stable. It is shown that the solutions of (1) are unbounded and tend to infinity as $t \rightarrow \infty$ if the matrix A is not stable.

The third part of the paper is devoted to the study of the asymptotic behavior of the solutions of the system (1) as $t \rightarrow \infty$ if the matrix A is not stable and the matrix B is not zero. It is shown that the solutions of (1) are unbounded and tend to infinity as $t \rightarrow \infty$ if the matrix A is not stable and the matrix B is not zero. The fourth part of the paper is devoted to the study of the asymptotic behavior of the solutions of the system (1) as $t \rightarrow \infty$ if the matrix A is not stable and the matrix B is zero. It is shown that the solutions of (1) are bounded and tend to zero as $t \rightarrow \infty$ if the matrix A is not stable and the matrix B is zero.

In gas-cooled reactors, the value of β may be quite large for the low-order resonance peaks and become smaller as the order increases. It is shown that the effect of resonance peaks is to increase the initial rate of reactivity response. The value of β may be quite large for the low-order resonance peaks and become smaller as the order increases. It is shown that the effect of resonance peaks is to increase the initial rate of reactivity response.

NUMERICAL SOLUTION

The analytical solution of equation (1) is not possible in general. A numerical solution is obtained by using the Runge-Kutta method. The numerical solution is compared with the analytical solution for the case where the matrix A is stable. It is shown that the numerical solution is in good agreement with the analytical solution. The numerical solution is also compared with the analytical solution for the case where the matrix A is not stable. It is shown that the numerical solution is in good agreement with the analytical solution.

The numerical solution of equation (1) is obtained by using the Runge-Kutta method. The numerical solution is compared with the analytical solution for the case where the matrix A is stable. It is shown that the numerical solution is in good agreement with the analytical solution. The numerical solution is also compared with the analytical solution for the case where the matrix A is not stable. It is shown that the numerical solution is in good agreement with the analytical solution.

compatible with the experimental equipment to be described later.

In Bankston's calculation the coolant flow selected was 10 cfm of standard air at a pressure of 75 psig. The power transient was 30.9 watts/inch. Bulk fluid properties were evaluated at 300°F and film properties for calculating the heat transfer coefficient were evaluated at 400°F. The heat transfer coefficient was evaluated from

$$\frac{hD}{k} = 0.23 \left(\frac{DG}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4} \quad 29$$

where new symbols are

D = inside tube diameter

k = thermal conductivity

G = mass velocity = w/A_f

μ = viscosity

From the above specifications, the following values for the similarity numbers were computed:

M = $3.88 \times 10^{-3} m$ where m is the metal temperature in °F;

F = $3.88 \times 10^{-3} f$ where f is the fluid temperature in °F;

X = 0.437 x where x is distance on the tube in feet;

T = 19.7 t where t is time in seconds; and

$$a = 265 = \frac{\delta_m c_m A_m}{\delta_f c_f A_f}$$

The relatively large value of "a" causes the temperature change during the first time interval to be only 2.88°F at $x = 6$ feet where the steady state temperature ($t \rightarrow \infty$) is 932°F. It is apparent that solution may begin with the

comparable with the experimental equipment to be described later. In addition, a calculation of the coolant flow rate was made on the basis of a pressure of 15 psi. The power transfer was 30.9 watts. Both fluid properties were evaluated at 300°F and fluid properties for calculating the heat transfer coefficient were evaluated at 400°F. The heat transfer coefficient was evaluated from

$$h = \frac{k}{D} \left(\frac{Pr}{Pr_s} \right)^{0.4} \left(\frac{\mu_s}{\mu} \right)^{0.14}$$

20

where the symbols are

- D = tube diameter
- k = thermal conductivity
- u = mean velocity = w/A_c
- μ = viscosity

From the above specifications, the following values for the fluid properties were computed:

- $k = 3.16 \times 10^{-5}$ where μ is the fluid viscosity in $lb/ft \cdot sec$
- $\mu = 5.03 \times 10^{-5}$ where μ is the fluid viscosity in $lb/ft \cdot sec$
- $\mu_s = 0.417$ where μ_s is viscosity at the tube inlet
- $\mu = 19.7$ where μ is viscosity in $lb/ft \cdot sec$

$$h = \frac{3.16 \times 10^{-5}}{0.01} \left(\frac{5.03 \times 10^{-5}}{0.417} \right)^{0.4} \left(\frac{19.7}{5.03 \times 10^{-5}} \right)^{0.14}$$

The relatively large value of h causes the temperature change during the first time interval to be only 1.33°F at $x = 2$ feet where the steady state temperature ($t = \infty$) is 300°F. It is apparent that solution may begin with the

equations for the second time interval and assumed initial metal and fluid temperatures of zero.

The convergence criteria for determining the size of Δx and Δt was not found but on the basis of judgement, convenience and an estimate of the error, the choice was $\Delta x = 1/2$ foot and $\Delta t = 5$ seconds.

The calculations used a finite difference rectangular grid in the X,T plane with nodes at intervals of ΔX and ΔT . The nodal temperatures represented the average temperature of a section of length ΔX during the time interval ΔT . The finite difference equations were derived from the non-dimensional differential equations 11 and 12, and involved both M and F at four adjacent nodes. The solution starts with the initial and boundary conditions which provide M and F at three of the nodes. The two finite difference equations provide M and F at the fourth node. The procedure was continued to $X = 2.622$ and $T = 2500$.

The method employed is straightforward and relatively simple but the actual work is long and tedious. Since each calculation depends on all previous calculations, computation errors are a serious threat. The major disadvantage of this type of numerical calculation is that the entire process may have to be repeated in order to demonstrate the effect of a new ratio of heat capacities, "a." Without convergence criteria the accuracy of the results is slightly uncertain.

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The results of this numerical calculation are compared with experimentally determined temperatures in a later section.

The results of this statistical analysis are shown in Table 1.

Experimentally determined temperature is a first-order

CHAPTER 4

EXPERIMENTAL INVESTIGATION

The theoretical investigation in the preceding chapter resulted in analytical and numerical solutions for the metal and bulk temperatures as functions of length and time. The necessary assumptions and complexity of the analytical solution and the time-consuming quality of the numerical solution suggest experimental methods. A thermal analog of the reactor coolant channel has the advantage that actual thermodynamic changes in properties are included. This may lead to fewer restrictions than required by the theoretical methods and give greater confidence in the results. A disadvantage is that construction of the model, data reduction, and analysis of results are also quite time consuming.

APPARATUS

The analog investigated here was a thin-walled metal tube which was heated by its resistance to the flow of electrical current. The coolant was air at 75 psig. Temperatures were detected by thermocouples placed at various places along the tube and at the coolant exit. Thermocouple outputs were recorded on an oscillograph. The power supply was a d.c. arc welder and an approximate step function change in heat generation was produced by completing the circuit with a knife switch.

An overall picture of the heated tube and associated equipment is shown in Figure 2. Details of the lower end of the heated tube are shown in Figure 3. This picture shows two

The first part of the paper is devoted to a description of the experimental apparatus and the method of measurement. It is shown that the apparatus is capable of measuring the rate of change of the concentration of a gas in a closed vessel with an accuracy of about 1%. The results of the measurements are given in Table I. It is seen from the table that the rate of change of the concentration of the gas is a function of the initial concentration of the gas and of the time interval between the measurements. The rate of change of the concentration of the gas increases with increasing initial concentration and with decreasing time interval.

APPENDIX

The author is indebted to Prof. Dr. A. J. V. van der Ziel for his valuable assistance in the preparation of this paper. The author is also indebted to Prof. Dr. J. H. van den Hul for his valuable assistance in the preparation of this paper. The author is also indebted to Prof. Dr. J. H. van den Hul for his valuable assistance in the preparation of this paper.

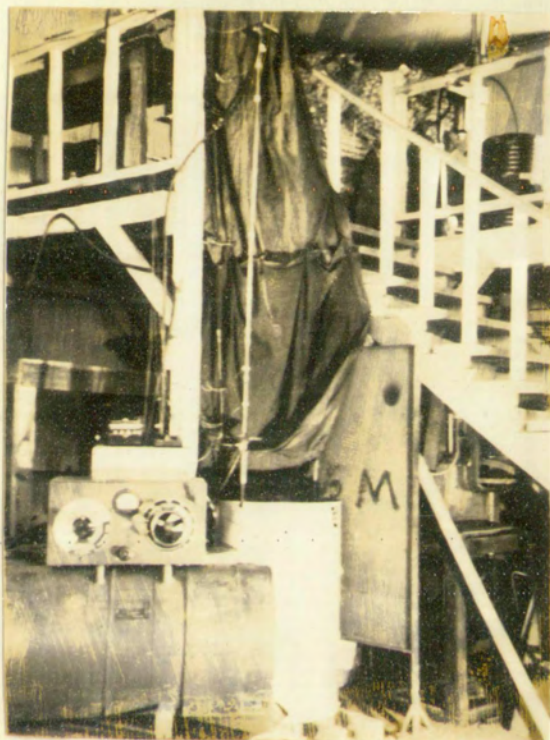
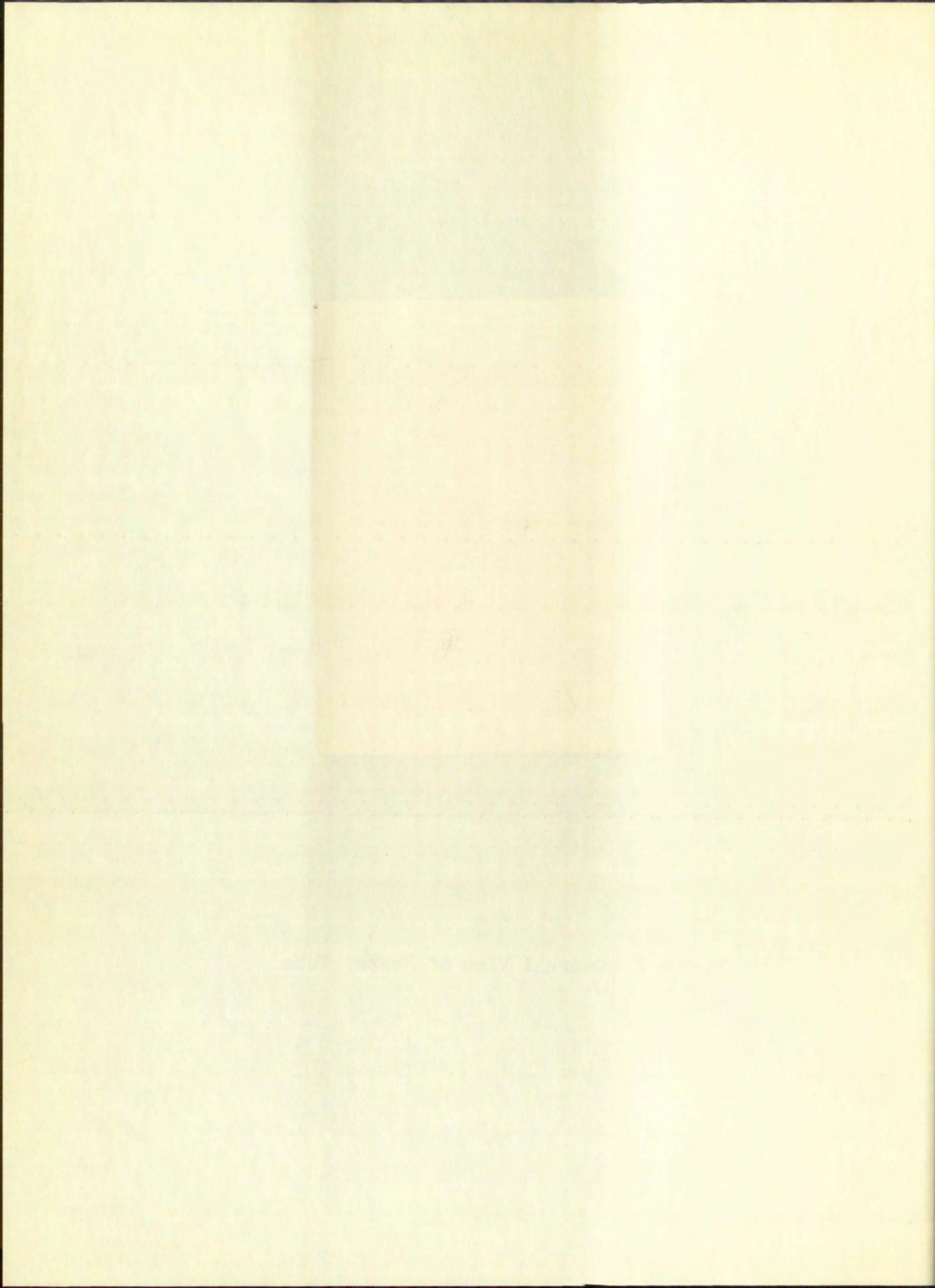


Figure 2 Overall View of Heated Tube



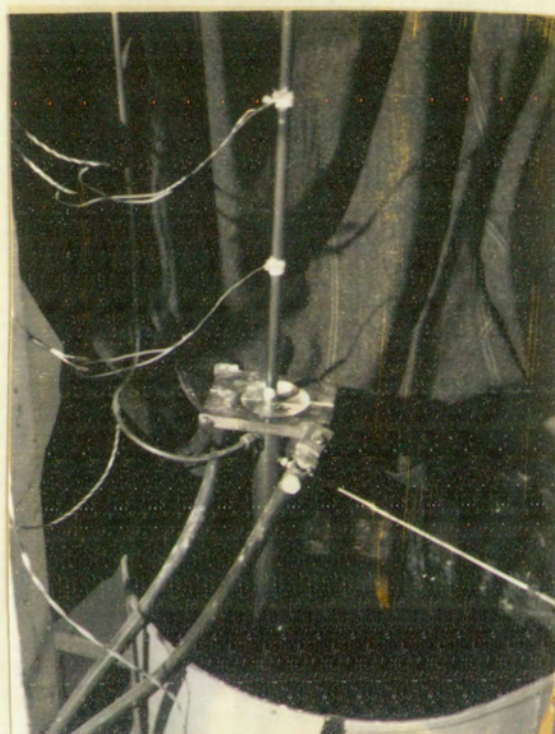


Figure 3 Detail of Electrode and Thermocouples

Figure 2 Detail of Electrode and Transducer

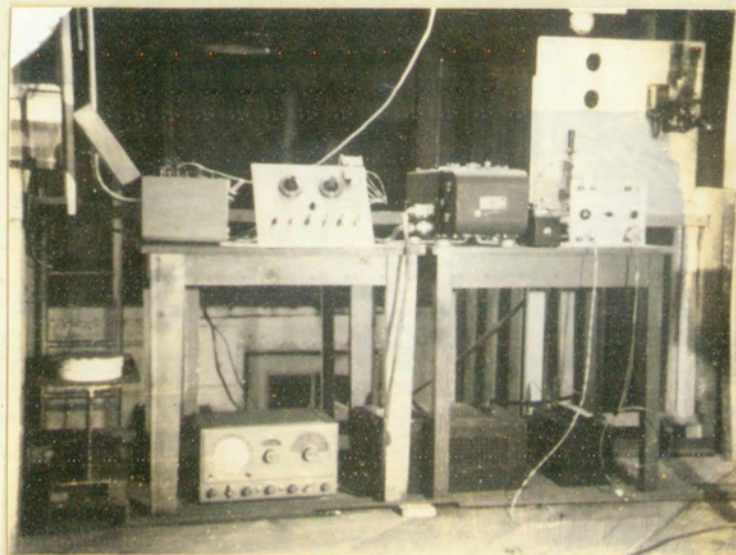
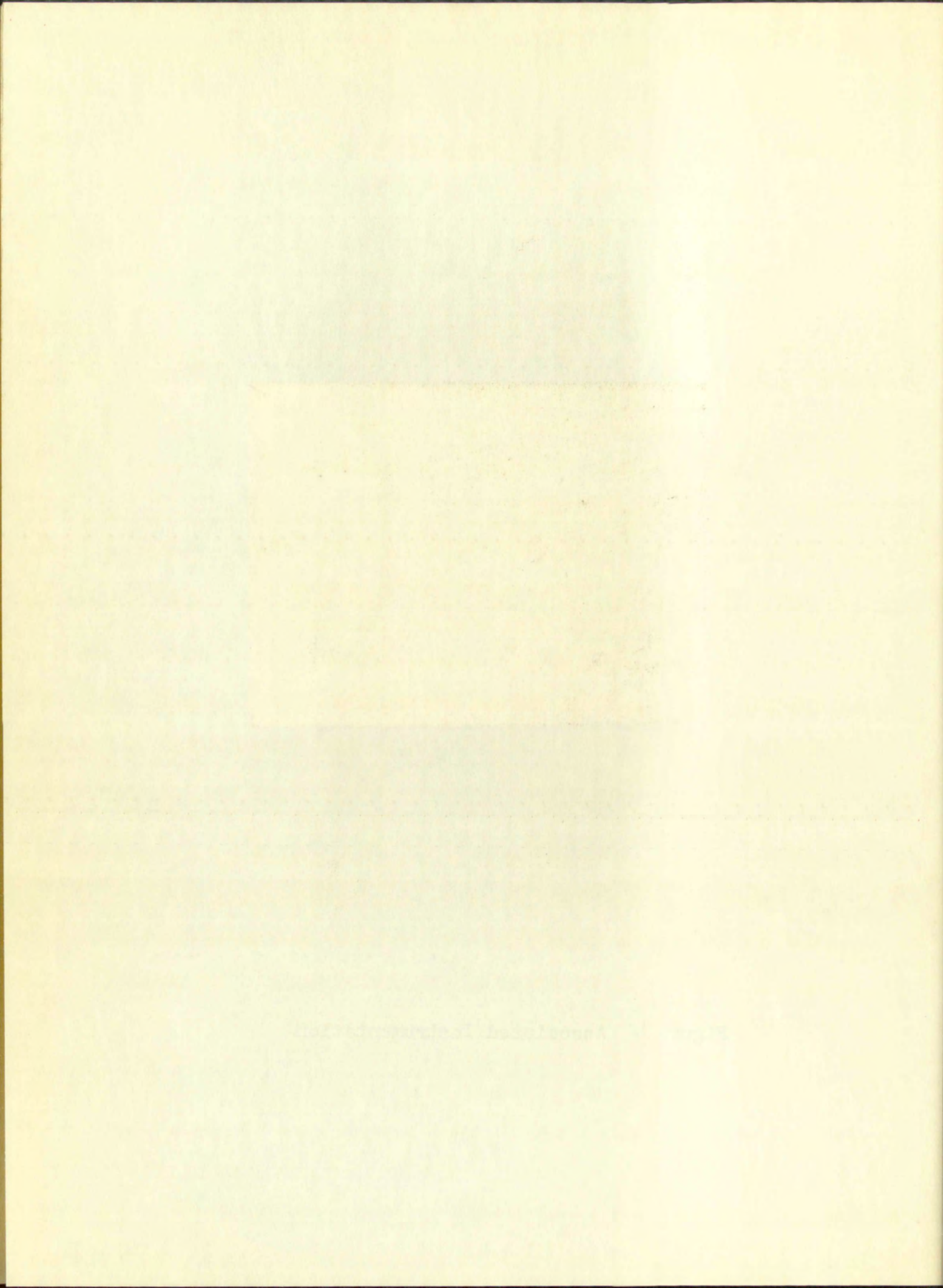


Figure 4 Associated Instrumentation



thermocouples installed on the tube, one of the electrical contacts, and the coolant temperature probe. Additional instrumentation is shown in an overall view in Figure 4.

The experimental data obtained were for three levels of heat generation and for three coolant flow rates. Other variables were maintained in agreement with the specification for the numerical solution. Four runs were made for the heat generation level and flow rate corresponding to the numerical solution specification. Two runs each were made of the other four experimental conditions.

Heat Exchanger -- The test section was an Inconel metal tube 6 feet long, 1/2 inch o.d. and 0.035 inch wall thickness. This simulated the reactor fuel element. Inconel was chosen because it has a relatively high electrical resistivity (100 μ - ohm-cm) which does not change appreciably with changes in temperature. It also has high strength properties at high temperatures.

A three foot length of tube at the inlet end was left unheated for the development of a stable turbulent flow pattern. This corresponds to $L/D \cong 85$ which is more than sufficient to minimize starting effects on the heat transfer coefficient.

Air System -- The coolant for the analog was air supplied by the compressed air system in the Mechanical Engineering shop. Two pressure regulators reduced and regulated the inlet air pressure to the heated tube. One of the regulators also incorporated an oil and water separator which removed most of the

The experimental data obtained in this work are presented in Table I. The results show that the rate of reaction is first order with respect to the concentration of the reactant and zero order with respect to the concentration of the catalyst. The rate constant, k , is found to be independent of the concentration of the catalyst, which is consistent with the proposed mechanism. The activation energy, E_a , is determined to be 15.2 kcal/mole. The pre-exponential factor, A , is found to be 1.2×10^6 sec⁻¹. The experimental data are fitted to the Arrhenius equation, $\ln k = \ln A - E_a/RT$, and the plot of $\ln k$ versus $1/T$ is shown in Figure 1. The linearity of the plot confirms the validity of the Arrhenius equation for this reaction. The rate of reaction is also found to be independent of the concentration of the solvent, which is consistent with the proposed mechanism. The experimental data are fitted to the rate equation, $r = k[A]$, and the plot of $\ln r$ versus $\ln [A]$ is shown in Figure 2. The linearity of the plot confirms the first-order dependence of the rate on the concentration of the reactant. The experimental data are also fitted to the rate equation, $r = k[A]^n$, and the plot of $\ln r$ versus $n \ln [A]$ is shown in Figure 3. The linearity of the plot confirms the first-order dependence of the rate on the concentration of the reactant. The experimental data are also fitted to the rate equation, $r = k[A]^n$, and the plot of $\ln r$ versus $n \ln [A]$ is shown in Figure 4. The linearity of the plot confirms the first-order dependence of the rate on the concentration of the reactant. The experimental data are also fitted to the rate equation, $r = k[A]^n$, and the plot of $\ln r$ versus $n \ln [A]$ is shown in Figure 5. The linearity of the plot confirms the first-order dependence of the rate on the concentration of the reactant.

moisture from the coolant, however, there was evidence of a slight amount of moisture in the test section.

The flow rate was determined by measuring the pressure drop across a sharp-edged orifice. The design of the orifice and its discharge coefficients were taken from the publication by Grace and Lapple(19). A 36-inch water manometer with a least division of 0.1 inch was used to obtain accurate differential pressure readings. Variations in flow rate during a run were observed to be on the order of 3%.

Inlet pressure to the test section was measured with a conventional pressure gauge. Pressure variations were on the order of 5%. Inlet air temperature was monitored by a mercury thermometer inserted in a well in the inlet piping.

The downstream end of the test section terminated in a 12 inch length of 1 inch stainless steel pipe. Access to this chamber was provided by two tapped holes; one located $3/4$ inch and the other about $8-3/4$ inch downstream from the end of the heated tube. The original intent of this chamber was to obtain a better measurement of the coolant bulk temperature by insuring mixing in an unheated region. Lack of time prevented utilizing the chamber except for the mounting of the coolant temperature probe in the upstream access hole.

Air leaving the mixing chamber passed through about 50 feet of coiled $1/2$ inch copper tubing which was immersed in a barrel of water. Thus, hot air leaving the analog was cooled to room temperature before reaching the flow-rate controlling valve at

...the coolant, however, there was evidence of a slight amount of leakage in the test section.

The flow rate was determined by measuring the pressure drop across a sharp-edged orifice. The design of the orifice and the aluminum manifolds were taken from the publication by Grace and Lapple (1953). A 30-inch water manometer with a least division of 0.1 inch was used to obtain accurate differential pressure readings. Variations in flow rate during a run were observed to be on the order of 2%.

Inlet pressure to the test section was measured with a conventional pressure gauge. Pressure variations were on the order of 0.2. Inlet air temperature was monitored by a thermistor inserted in a well in the inlet piping.

The downstream end of the test section terminated in a 12 inch length of 1 inch stainless steel pipe. Access to this chamber was provided by two tapped holes; one located 3/4 inch and the other about 5-3/4 inch downstream from the end of the heated tube. The original intent of this chamber was to obtain a better measurement of the coolant bulk temperature by insuring mixing in an unheated region. Lack of time prevented utilizing the chamber except for the monitoring of the coolant temperature probe in the upstream access hole.

After leaving the mixing chamber passed through about 20 feet of coiled 1/2 inch copper tubing which was immersed in a barrel of water. Thus, after leaving the mixing chamber cooled to room temperature before reaching the flow-rate measuring valve at

the end of the system.

Electrical Power System -- The electrical power circuit diagram is shown in Figure 14 in the Appendix.

Power for heating the analog was provided by a DC welder power supply. Since this was an AC-DC motor-generator set, isolation from ground was provided. Current measurement was accomplished with a new 50 mv.-200 ampere shunt and a 0-100 millivoltmeter with a least division of one millivolt. Voltage was measured with a 0-30 voltmeter with a least division of 0.5 volts. Both meters were calibrated against a standard cell prior to the experiments.

Circuit closing was done with a knife-switch. The circuit was opened with a circuit breaker which also provided protection against accidental shorts. Difficulty was experienced in obtaining a consistent power level from the generator. This was overcome by providing an attendant to continually adjust the welder controls during the experimental runs. Some power fluctuation resulted but the effect is significant only during the first few seconds of each run.

An effort was made to nullify the effect of the heat capacity associated with the electrical contacts on the tube. The contactor developed was intended to be self-heating so that there would be a negligible temperature gradient between the tube and the contact and hence no heat loss.

The end of the system.

Electrical Power Input -- The electrical power input

Diagram is shown in Figure 1 in the Appendix.

Power for heating the anode was provided by a 20-watt

power supply. Since this was an AC-DC motor-generator set,

isolation from ground was provided. Ground connection was

accomplished with a low 50 mv., 500 ohm resistor and a 0-100

millivoltmeter with a least division of one millivolt. Voltage

was measured with a 0-30 voltmeter with a least division of 0.5

volts. Both meters were calibrated against a standard cell

prior to the experiment.

Circuit diagram was done with a knife-switch. The circuit

was opened with a circuit breaker which also provided protec-

tion against accidental shorts. Difficulty was experienced in

obtaining a constant power level from the generator. This was

overcome by providing an attendant to continually adjust the

control knobs during the experimental run. Gas power

fluctuation varied but the effect is significant only during

the first few seconds of each run.

An effort was made to nullify the effect of the heat

capacity associated with the electrical contacts on the tube.

The contact developed was intended to be self-heating so that

there would be a negligible temperature gradient between the

tube and the contact and hence no heat loss.

A view of this contactor is shown in Figure 3. A 3 inch disc of Incoloy with a concentric hole of 1/4 inch radius was machined to a thickness of 1/32 inch for all radii greater than 1/2 inch. The thickness at the edge of the hole which contacted the tube was 1/8 inch. Between the edge of the hole and the 1/2 inch radius, the thickness decreased uniformly to 1/32 inch. The disc was then split across a diameter and the two halves soldered to copper blocks. In use, the two halves of the disc were assembled around the Inconel tube and clamped in place by studs through the copper blocks. Electrical leads were attached to the copper so that current would flow through the disc to the tube and the disc would be heated by the I^2R loss within it.

The effectiveness of this contact was not fully investigated.

Thermocouples -- Chromel-alumel thermocouples were chosen for measuring tube temperatures at the lower end of the tube because of their suitability for high temperature applications. Because of the superior thermal e.m.f., iron-constantan thermocouples were used for the three measurements on the upper portion of the tube where lower temperatures were expected. All thermocouples were made from 26 gage wire.

The attachment of the thermocouples to the tube was the subject of some investigation. Attempts to provide a metallic bond failed because of difficulties in welding the dissimilar materials and because of stray electrical effects. The following technique was finally used: First, a thin layer of seriesen

A view of this contact is shown in Figure 3-4. The disc of Incoloy with a diameter hole of 1/8 inch was mounted to a thickness of 1/8 inch for all wall greater than 1/2 inch. The thickness at the edge of the hole which contacted the tube was 1/8 inch. Between the edge of the hole and the 1/2 inch radius, the thickness decreased uniformly to 1/8 inch. The disc was then split across a diameter and the two halves adhered to copper blocks. In use, the two halves of the disc were assembled around the Inconel tube and clamped in place by bands through the copper blocks. Electrical leads were attached to the copper so that current would flow through the disc to the tube and the disc would be heated by the I²R loss within it.

The effectiveness of this contact was not fully investigated. The thermocouples -- Chromel-constantan thermocouples were chosen for measuring tube temperatures at the lower end of the tube because of their stability for high temperature applications. Because of the superior thermal e.m.f. iron-constantan thermocouples were used for the three radiometers on the upper portion of the tube where lower temperatures were expected. All thermocouples were made from 36 gage wire. The attachment of the thermocouples to the tube was the subject of some investigation. Attempts to provide a seal-off had failed because of difficulties in making the distal portion and because of stray electrical effects. The following technique was finally used: First, a thin layer of acetone

ceramic cement was applied in a half-inch-wide strip three-quarters of the way around the tube. Each wire of the thermocouple pair was placed on the tube so that it was electrically insulated by the ceramic layer. The properly cleaned ends were twisted tightly and welded together. The joint was clipped so as to leave about 1/8 inch of the welded junction. This junction was then bent up to lay against the tube. A strip of 1/2 inch glass tape was tightly wrapped twice around the tube over the thermocouple joint and tied at the back. A piece of 26 gauge wire was wrapped once around the tube over the junction and fastened by twisting the ends together at the back. The wire held the thermocouple tightly against the tube.

Impedance matching considerations required that the thermocouple circuit have a minimum electrical resistance. Since the thermocouple materials have relatively high resistance, only a minimum length was used. Rough calculations indicated that the end of a six-inch length would not be affected by heat conduction from the hot junction. Copper leads were used beyond the end of the six inch length and the lead-to-thermocouple splice became the reference junction. The reference junction temperature was assumed to be at room temperature.

The adequacy of the thermocouple installation was demonstrated by steady state tests using a thermometer well clamped on to the tube over the thermocouple. Temperature readings were compared at equilibrium and during the cooling period

following removal of power. Excellent agreement of the temperatures indicated by the thermocouple and the thermometer was observed. This data is shown in Table 1 in the Appendix.

Fluid temperature was detected by means of an iron-constantan thermocouple probe constructed by Pratt and Whitney Aircraft Co. The probe consists of a 1/8 inch diameter stainless steel tube which encases the thermocouple wire. The probe was installed in the upper access hole of the mixing chamber. It was inserted so that the thermocouple junction was centered in the stream of air leaving the heated tube.

Oscillograph -- The output e.m.f. of the various thermocouples was detected by the galvanometers in a Hathway Type S-12A Oscillograph. The galvanometer traces were recorded on photographic paper moving at a speed of 1/2 inch/sec. A time marker system within the oscillograph provided marks on the paper at intervals of 1/10 second so that the time dependence of all temperatures could be accurately determined. The time marker was compared to a 60 cycle/sec signal and found to be extremely accurate.

CALIBRATION

Independent calibration of the individual galvanometers was not possible since they have a relatively small internal impedance (about 7 ohms). When connected to a thermocouple the power developed within them is greatly influenced by the impedance of the external circuit. This made it necessary to

calibrate the oscillograph galvanometer while they were connected with their thermocouples. To accomplish this, a switching circuit was devised so that a known e.m.f could be applied in series with the galvanometer-thermocouple circuit. A portion of the switching circuit is shown schematically in Figure 15 in the Appendix. This part is typical of the six independent circuits required for the six thermocouples.

The source of known e.m.f. was obtained by constructing a voltage divider circuit. The schematic diagram of this calibrating circuit is shown in Figure 16 in the Appendix. A 12 volt storage battery was used as the prime source of electrical power. Ten voltage steps were provided by using five taps in a series resistance circuit and a high-low range switch.

The actual output voltage of this circuit would depend upon the output impedance. Since each galvanometer-thermocouple circuit has a different impedance, it was necessary to put the circuit to be calibrated into a leg of a Wheatstone bridge. The bridge is balanced by adjusting resistors which shunt the galvanometer circuit. The voltage divider then feeds the bridge which, when balanced, always constitutes the same impedance. Since the other three legs of the bridge are fixed, the voltage across the galvanometer is also fixed.

The voltage corresponding to each step was determined by comparison on a Leeds and Northrup No. 8662 Portable Precision Potentiometer.

calculated the overall galvanometer voltage for each
connected with each thermopile. To accomplish this,
a balancing circuit was devised so that a known e.m.f. could be
applied in series with the galvanometer-thermopile circuit.
A portion of the balancing circuit is shown schematically in
Figure 12 in the Appendix. This part is typical of the six
independent circuits required for the six thermopiles.
The source of known e.m.f. was obtained by connecting a
voltage divider circuit. The characteristic diagram of this cell-
balancing circuit is shown in Figure 13 in the Appendix. A 12
volt standard battery was used as the prime source of electrical
power. Ten voltage steps were provided by using five pairs in a
series resistance circuit and a high-low range switch.
The actual output voltage of this circuit would depend
upon the output impedance. Since each galvanometer-thermopile
circuit has a different impedance, it was necessary to put the
circuit to be calibrated into a leg of a Wheatstone bridge.
The bridge is balanced by adjusting resistors which show the
galvanometer circuit. The voltage divider then feeds the
bridge which, when balanced, always connected the same impe-
dance. Since the other three legs of the bridge are fixed, the
voltage across the galvanometer is also fixed.
The voltage corresponding to each step was determined by
connection on a Leeds and Northrup No. 9663 portable resistance
potentiometer.

The resistance of the fluid temperature probe was about the same magnitude as the resistance of the galvanometer. It was evident that only half of the power resulting from the thermal e.m.f. would be developed in the galvanometer. This was overcome by the use of a low-level current amplifier designed by Summers(20) which matched impedances and provided some amplification. A schematic of this amplifier is presented in Figure 17 in the Appendix.

After carefully determining the output voltages for each step of the calibrating circuit, the galvanometer-thermocouple circuits were calibrated in turn with the corresponding galvanometer deflection being recorded on the oscillograph record. A typical calibration plot of deflection versus applied e.m.f. is shown in Figure 18 in the Appendix. It will be noted that the response is essentially linear over the range of interest. The assumption that the e.m.f. output of the thermocouples used is directly proportional to temperature is within the accuracy required here. Then the conversion from galvanometer trace deflection to temperature is simply $\text{temperature} = K(\text{deflection})$ where K must be determined for each circuit. The K's determined are tabulated in Table 2 in the Appendix.

EXTERNAL HEAT LOSSES

The external surface of the heated tube was not insulated and heat transferred from this surface was found to be appreciable. This loss was found experimentally by measuring the equilibrium temperature at each thermocouple location when a

The first part of the paper deals with the general theory of the problem. It is shown that the problem can be reduced to a set of linear equations. The solution of these equations is given in the Appendix.

After a preliminary discussion of the problem, the author proceeds to a detailed analysis of the various cases. It is shown that the problem can be solved in a number of cases. The results are given in the Appendix.

APPENDIX

The Appendix contains the detailed solution of the linear equations mentioned in the text. It is divided into several parts, each dealing with a different case.

measured amount of power was delivered to the tube and there was no coolant flow. Under these conditions, essentially all of the input power is dissipated at the outer tube surface. Conduction loss at the ends is quite small since the cross-section is small. Data from this experiment is presented in Table 3 in the Appendix. A plot of surface temperature versus heat loss is shown in Figure 19 in the Appendix. Curves are presented for several distances down the tube from the inlet end. Heat loss is somewhat greater at the upper end of the tube because of the greater natural convection velocity.

During this experiment, the thermal e.m.f. was measured on the precision potentiometer and was also recorded on the oscillograph. This provided an additional check on galvanometer calibration and is indicated in Figure 18 in the Appendix.

EXPERIMENTAL TESTS

An outline of the procedure followed in performing an experimental run follows: The coolant flow conditions were carefully set up. The oscillograph was started and the knife-switch closed to start heat generation in the tube. Galvanometer traces were observed on the viewing screen of the oscillograph and were photographically recorded. When no further deflection could be detected, the oscillograph was shut off. The portable precision potentiometer was connected to the calibrating input terminals of the thermocouples switching circuit (Figure 15) and the thermal e.m.f. was measured

The general nature of the work was described in the first part of the report. Under these conditions, essentially all of the heat loss is dissipated at the lower tube surface. Construction loss at the ends is quite small since the cross-section is small. Data from this experiment is presented in Table 2 in the Appendix. A plot of surface temperature versus heat loss is shown in Figure 19 in the Appendix. Curves are presented for several distances from the tube from the inlet end. Heat loss is somewhat greater at the upper end of the tube because of the greater natural convection velocity.

During this experiment, the thermal conductivity was measured on the precision potentiometer and was also recorded on the oscillograph. This provided an additional check on natural convection and is indicated in Figure 18 in the Appendix.

EXPERIMENTAL TESTS

An outline of the procedure followed in performing the experiments was follows: The coolant flow conditions were carefully set up. The oscillograph was started and the inlet section closed to start heat generation in the tube. Galvanometer traces were observed on the viewing screen of the oscillograph and were photographically recorded. When no further deflection could be obtained, the oscillograph was shut off. The potentiometer potentiometer was connected to the calibration figure terminals of the thermocouple within the circuit (Figure 18) and the thermal conductivity was measured.

directly for each thermocouple. Input power was recorded.

A total of six experimental tests were made. Two runs were made for each test.

The inlet air pressure was 75 psig in all of the tests. This results in a similarity number "a" of 265 and corresponds to the "a" used in computing the numerical solution. The tests involved significant variations in the input power and air flow rate only. The following is a summary of the experimental conditions for each test:

Test No. 1, $w = 0.865$ lb/min, input power = 30.9 watts/in.

Test No. 2, $w = 0.708$ lb/min, input power = 30.9 watts/in.

Test No. 3, $w = 0.500$ lb/min, input power = 30.9 watts/in.

Test No. 4, $w = 0.865$ lb/min, input power = 13.9 watts/in.

Test No. 5, $w = 0.865$ lb/min, input power = 41.6 watts/in.

Test No. 6, $w = 0.865$ lb/min, input power = 30.9 watts/in.

DATA REDUCTION

After developing the oscillograph record, selected times after the beginning of the power transient were located. A set of proportional dividers were set up to give the proper conversion from deflection to temperature as indicated by Table 2. The temperature from each thermocouple circuit was determined for the selected times and recorded. The compilation of these temperatures is presented in Tables 4 and 5 in the Appendix.

The averaged data from Table 4 was plotted on graph paper as temperature versus distance on the tube. Smooth curves were drawn through points for the same time after the beginning

divided for each thermocouple. The total of six experimental runs were made for each test.

The inlet air pressure was 27.5 psia at all times. This results in a stability number of 0.05 and a Mach number of 0.2. The use of nitrogen as the working fluid was chosen for the reasons involved in the previous sections. The flow rate was 1.0 lb/min. The following is a summary of the experimental conditions for each test:

- Test No. 1, $\dot{m} = 0.055$ lb/min, input power = 20.0 watts
- Test No. 2, $\dot{m} = 0.070$ lb/min, input power = 20.0 watts
- Test No. 3, $\dot{m} = 0.200$ lb/min, input power = 20.0 watts
- Test No. 4, $\dot{m} = 0.055$ lb/min, input power = 15.0 watts
- Test No. 5, $\dot{m} = 0.055$ lb/min, input power = 15.0 watts
- Test No. 6, $\dot{m} = 0.055$ lb/min, input power = 15.0 watts

DATA REDUCTION

After developing the calibration curves, the data were reduced after the beginning of the power transient had passed. A set of proportional dividers were set up to give the proper correction. Also from Section 2, the temperature as indicated by Table 1. The temperature from each thermocouple circuit was corrected for the selected time and term. The correction of these temperatures is presented in Table 1 and 2 in the Appendix. The averaged data from Table 1 was plotted on graph paper as temperature versus distance on the x-axis. The resulting curves from these points for each test are shown in the following

of the transient. The purpose of "smoothing" the data was to minimize scattering and eliminate erratic readings. A typical example of this procedure is shown in Figure 20 in the Appendix where the smoothed curves for Test 2 are presented.

The conversion to dimensionless temperature, time and distance was required next. The values for thermal properties used in the numerical solution were assumed to be adequate. The heat transfer coefficient was previously evaluated from

$$\frac{hD}{k} = 0.23 \left(\frac{DG}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4} \quad 29$$

where $G = w/A_f$. A flow rate correction for h can be made on the basis of equation 29 which yields the relation

$$\frac{h_1}{h_2} = \left(\frac{w_1}{w_2} \right)^{0.8} \quad 30$$

The proper ΔU_g to use in computing the dimensionless temperatures is determined by

$$\Delta U_g = (\text{input power}) - (\text{external power loss}) \quad 31$$

where the external power loss was determined from the "36 inch" curve of Figure 19 using the "smoothed" temperature at the location of interest. This technique is somewhat arbitrary but it may be argued that considerably more data and analytical evidence would be required to justify a different procedure.

The non-dimensional data may now be calculated using the above modifications. The resulting similarity numbers, along

at the standard. The purpose of "averaging" the data was to
 minimize the error and eliminate erratic readings. A typical
 example of this procedure is shown in Figure 10 in the appendix
 where the recorded curves for Test 2 are presented.

The conversion to dimensionless temperature, time and dis-
 tance was performed next. The values for thermal properties
 used in the theoretical solution were assumed to be constants.
 The heat transfer coefficient was previously evaluated from

$$h = \frac{q}{A_s (T_s - T_f)} = \frac{0.001 \text{ W}}{0.001 \text{ m}^2 (100 - 20) \text{ K}} = 0.0125 \text{ W/m}^2\text{K}$$

where $h = W/m^2K$. A flow rate correction for h can be made on
 the basis of equation 19 which yields the relation

$$h' = h \left(\frac{v'}{v} \right)^{0.8} = 0.0125 \left(\frac{0.001}{0.001} \right)^{0.8} = 0.0125 \text{ W/m}^2\text{K}$$

The proper h' to use in computing the dimensionless
 temperatures is determined by

$$h'_{ext} = (input\ power) - (external\ power\ loss)$$

where the external power loss was determined from the "0.5 inch"
 curve of Figure 19 using the "averaged" temperature at the
 location of interest. This technique is somewhat arbitrary but
 it can be argued that considerably more data and analysis
 evidence would be required to justify a different procedure.
 The one-dimensional case may now be calculated using the
 above conditions. The resulting statistical numbers along

with the "smoothed" temperature and external heat loss data are tabulated in Table 6 in the Appendix.

The measured fluid temperatures may be converted to non-dimensional form by a similar technique. In this case, fluid temperature was measured only at $x = 72$ inches. The external heat loss for correcting ΔU_g was obtained by using a temperature obtained by extrapolating the smoothed metal temperature to $x = 72$ inches and using the curve for "36 inches" in Figure 19 .

ERRORS

Estimates of the response time of the thermocouples was facilitated by information found in a text by Giedt(21). By extrapolation of data presented in his Table 14.1, the response of the coolant thermocouple was estimated to be $t = 0.3$ seconds where t is the time required for a thermocouple to reach 63.2% of a step change in temperature. The response of the tube thermocouples can be computed from

$$t = Wc/hA$$

32

where t = time to reach 63.2% of a step change, hr.

W = weight of the thermocouple, lbs.

$$= \gamma L \pi d^2 / 4$$

c = specific heat, BTU/lb^oF

h = coefficient of heat transfer, BTU/h f² °F

A = surface area of thermocouple, f²

$$= \pi dL.$$

with the "residual" temperature and residual heat loss rate
 tabulated in Table 6 in the Appendix.
 The measured fluid temperatures may be converted to true
 dimensional form by a similar technique. In order to find
 temperature was assumed only at a 1/2 inch interval.
 heat loss for correcting A_2 was obtained by using a temperature
 obtained by extrapolating the residual heat loss rate
 $x = 1/2$ inches and using the curve for 1/2 inch in Figure 10.

RESULTS

Estimates of the response time of the thermocouple are
 facilitated by information found in a paper by Kestel et al.
 extrapolation of data presented in his Table 1. The response
 of the coolant thermocouple was estimated to be 0.15
 seconds where t is the time required for a thermocouple to
 reach 63.2% of a step change in temperature. The response of
 the tube thermocouple can be computed from

$$t = W/C_p \Delta T$$

where t = time to reach 63.2% of a step change in
 W = weight of the thermocouple, lbs.
 C_p = specific heat, Btu/lb-F
 ΔT = coefficient of heat transfer, Btu/lb-F-in
 A = surface area of thermocouple, in²
 $t = 1.5$

This reduces to $t = \delta dc/4h$. The following values were assumed:

$$\delta = 540 \text{ lb/f}^3$$

$$d = 0.016 \text{ in.}$$

$$c = 0.11 \text{ BTU/lb } ^\circ\text{F}$$

$$h = 250 \text{ BTU/h f}^2 \text{ } ^\circ\text{F}$$

Substitution of these values results in $t = 7.9 \times 10^{-5} \text{ hr} = 0.28$ seconds.

Trace deflections were measured to an accuracy of about $\pm 1/64$ inches. The data reduction error = $1/64K$, ranges from ± 4 to ± 18 $^\circ\text{F}$.

This column is a δ value. The following values were

obtained:

$$\delta = 0.011 \text{ cm}^2$$

$$\delta = 0.011 \text{ cm}^2$$

$$\delta = 0.011 \text{ cm}^2$$

$$\delta = 0.011 \text{ cm}^2$$

Integration of these values results in a δ of 10^{-10} cm.

seconds.

These deflections were mounted to an accuracy of about

1 μ m inches. The data reduction error = 1.0%, ranges from

1 to 20 μ m.

CHAPTER 5

RESULTS

A summary of the results of Bankston's numerical solution is presented in Figure 5 by a graph of the similarity number M versus X for several values of T . This graph provides the necessary information for making a comparison between theory and analog results.

Tests 4, 5, and 6 investigated the effect of varying the magnitude of ΔU_g . The results of this series of tests is compiled in non-dimensional form in Table 6. The correlation of this data is shown in Figures 6, 7, and 8 where non-dimensional metal temperature, M , is plotted versus non-dimensional time, T . The three figures are for three particular values of non-dimensional distance, X .

According to the dimensional analysis, the effect of varying the flow rate should be accounted for by the similarity number X . This was investigated by Tests 1, 2, and 3. The non-dimensional data is also compiled in Table 6. Figures 9, 10 and 11 are plots of M versus T for tests 1, 2, and 3 respectively. Each graph contains the curves for three values of X . Superimposed on the experimental curves are the theoretical curves for the same values of X . These theoretical curves are drawn from information presented by Figure 6 as mentioned above.

The averaged fluid temperatures from Tests 1 and 6 were converted to non-dimensional temperature, F , and compiled in Table 7 in the Appendix. Figure 12 is a plot of F versus T

CHAPTER 7
RESULTS

A summary of the results of the present investigation is presented in Figure 2 by a graph of the average rate of reaction R versus X for several values of T . This graph provides the necessary information for making a comparison with the results of other investigators.

Tests 1, 2, and 3 investigated the effect of varying the magnitude of u_0 . The results of this series of tests are compared in non-dimensional form in Table 6. The correlation of this data is shown in Figure 6, 7, and 8 where X is plotted versus non-dimensional time t . The curves shown are for three particular values of u_0 and X .

According to the dimensional analysis, the effect of varying the flow rate should be accounted for by the parameter number X . This was investigated by Tests 1, 2, and 3. The non-dimensional data is also compiled in Table 6. Figures 7 and 8 are plots of R versus T for Tests 1, 2, and 3 respectively. Each graph contains the curves for three values of X . Superimposed on the experimental curves are the theoretical curves for the same values of X . These theoretical curves are drawn from information presented in Figure 6 and are labeled above.

The averaged fluid temperatures from Tests 1 and 2 were converted to non-dimensional temperatures T and plotted in Table 7 in the Appendix. Figure 10 is a plot of R versus T .

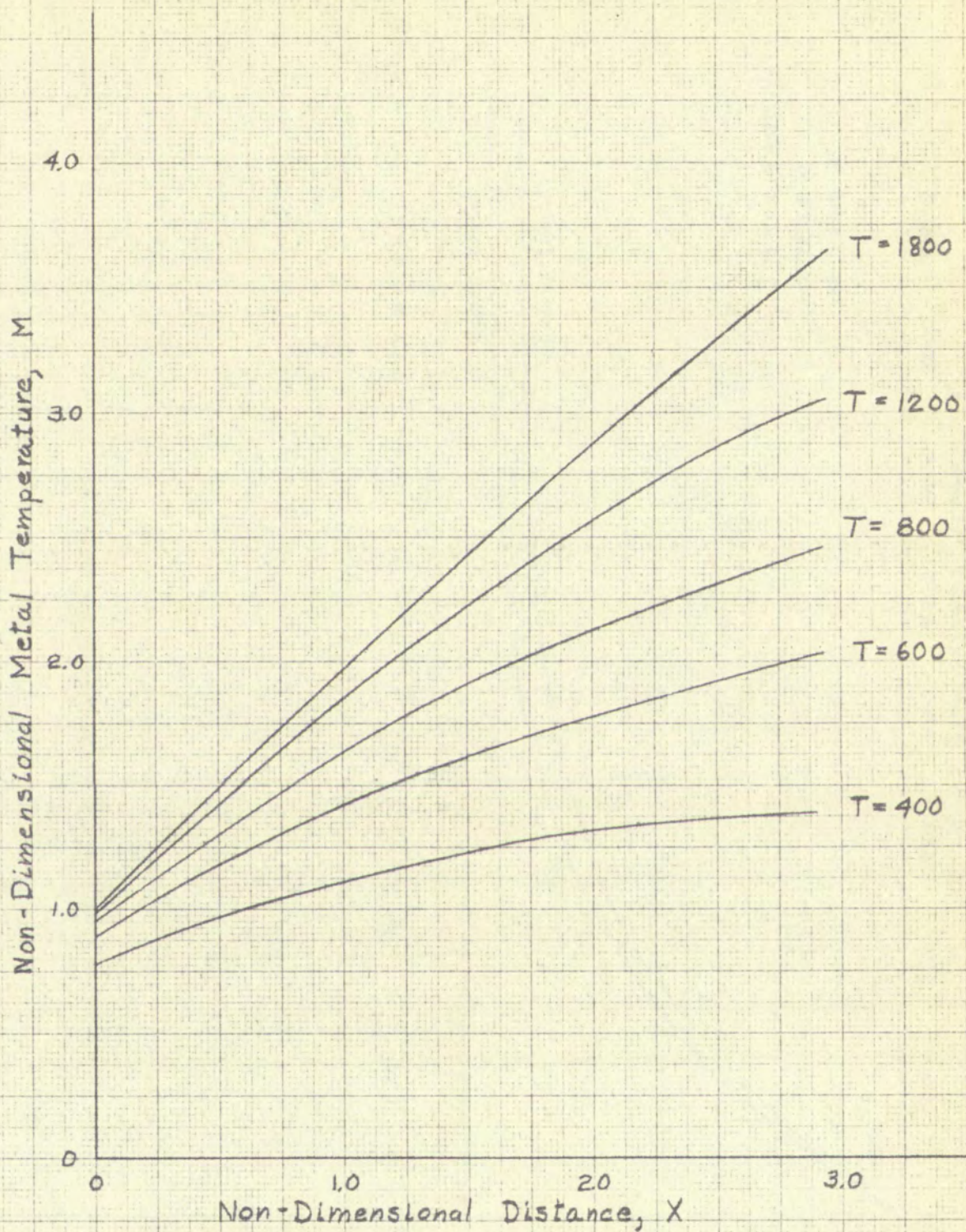
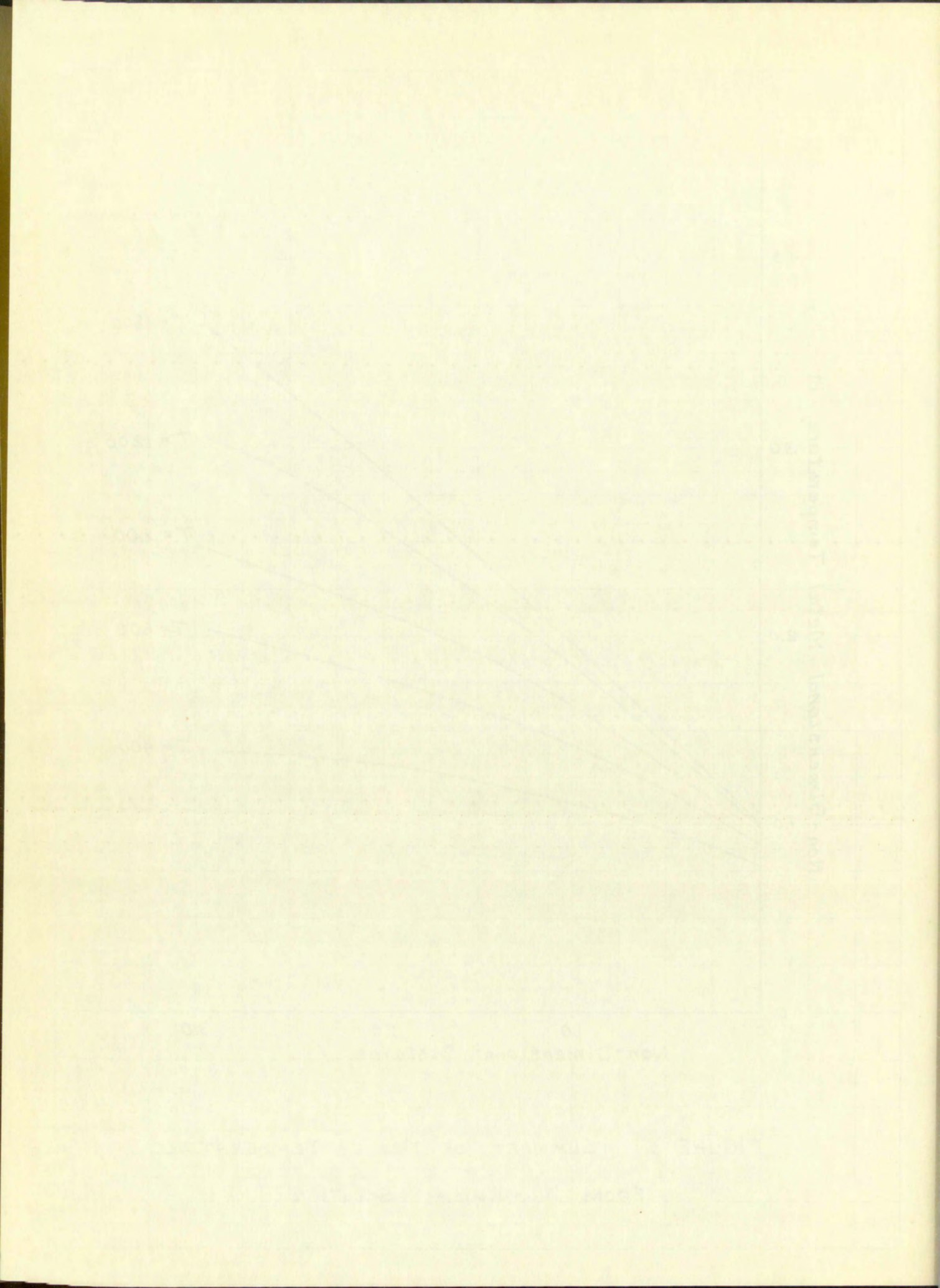


FIGURE 5 SUMMARY OF METAL TEMPERATURE
FROM NUMERICAL SOLUTION



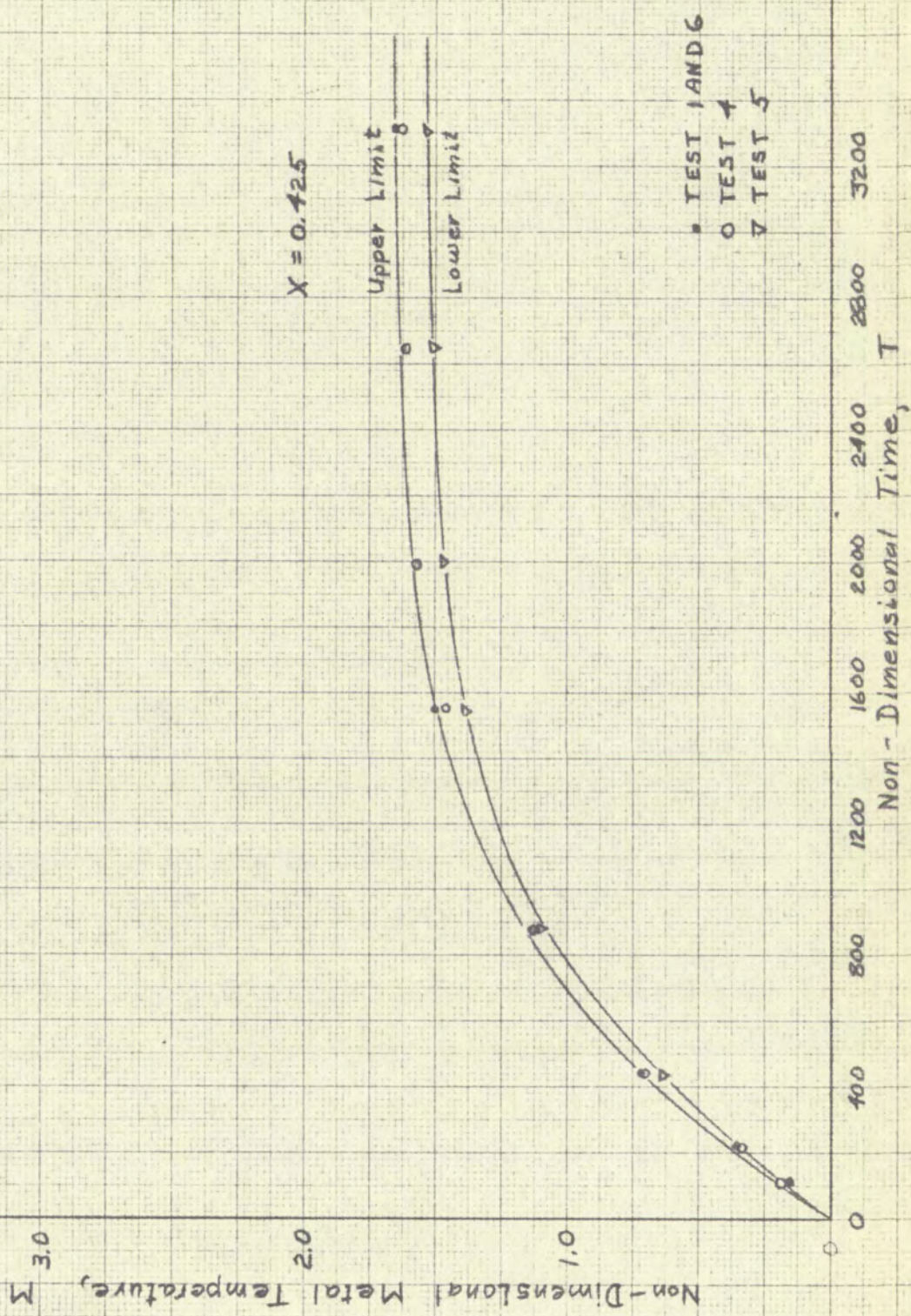
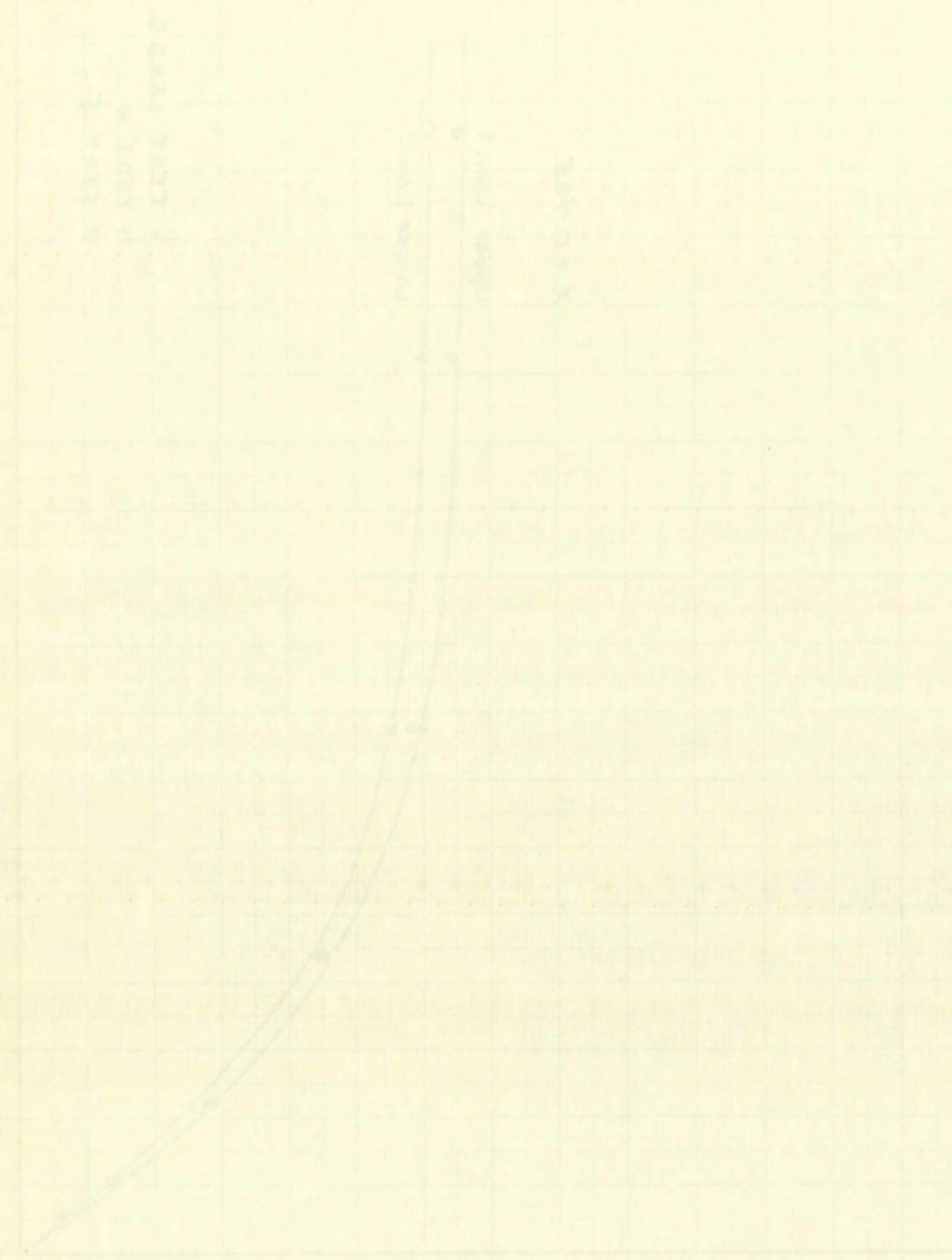


FIGURE 6 NON-DIMENSIONAL TEMPERATURE VS. TIME FOR TESTS 4, 5, 6

LIGHT TRANSMISSION THROUGH WATER

1.000 0.500 0.250 0.125 0.0625 0.03125 0.015625 0.0078125 0.00390625 0.001953125



0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0

0 10 20 30 40 50 60 70 80 90 100

TRANSMISSION

DEPTH IN METERS

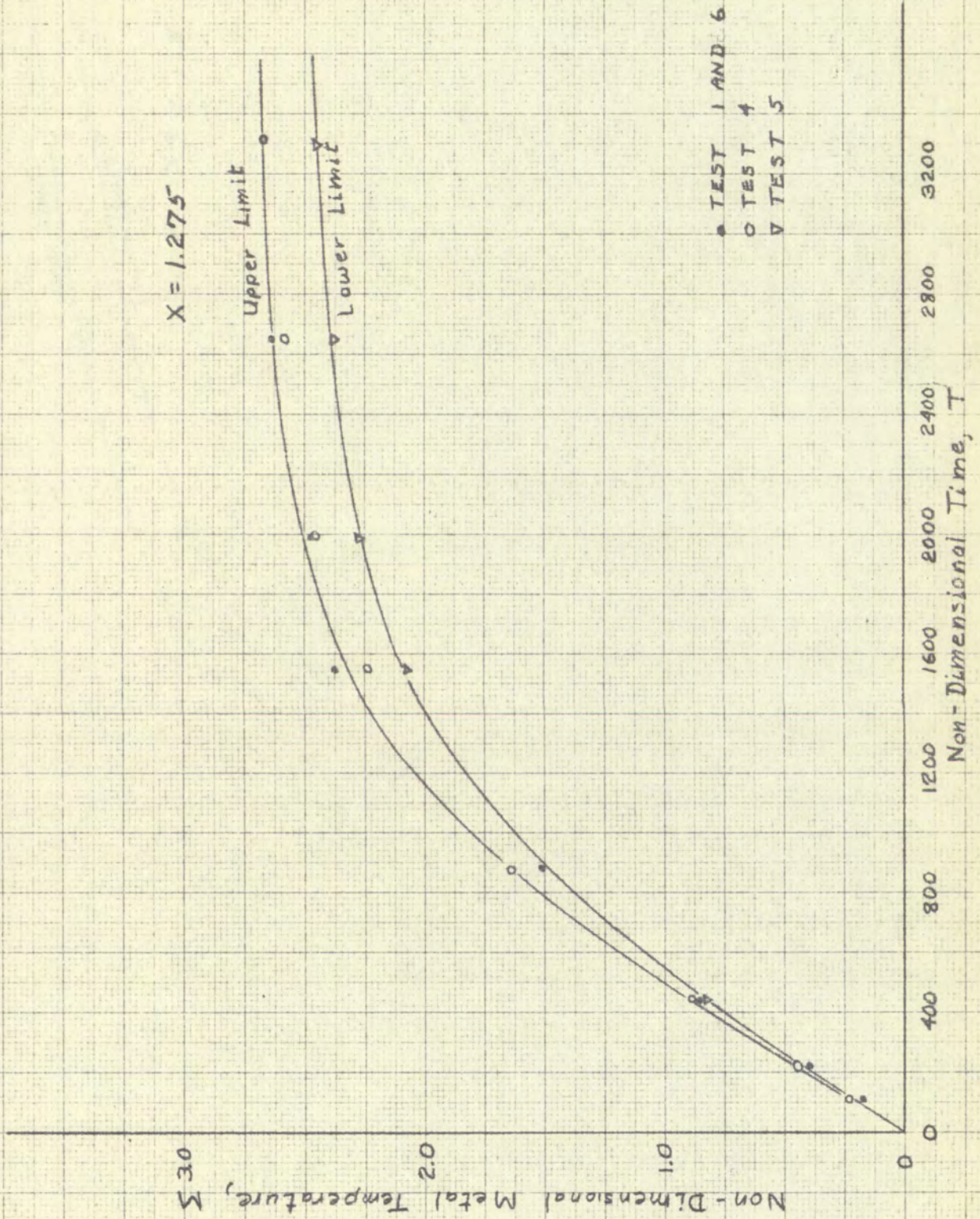
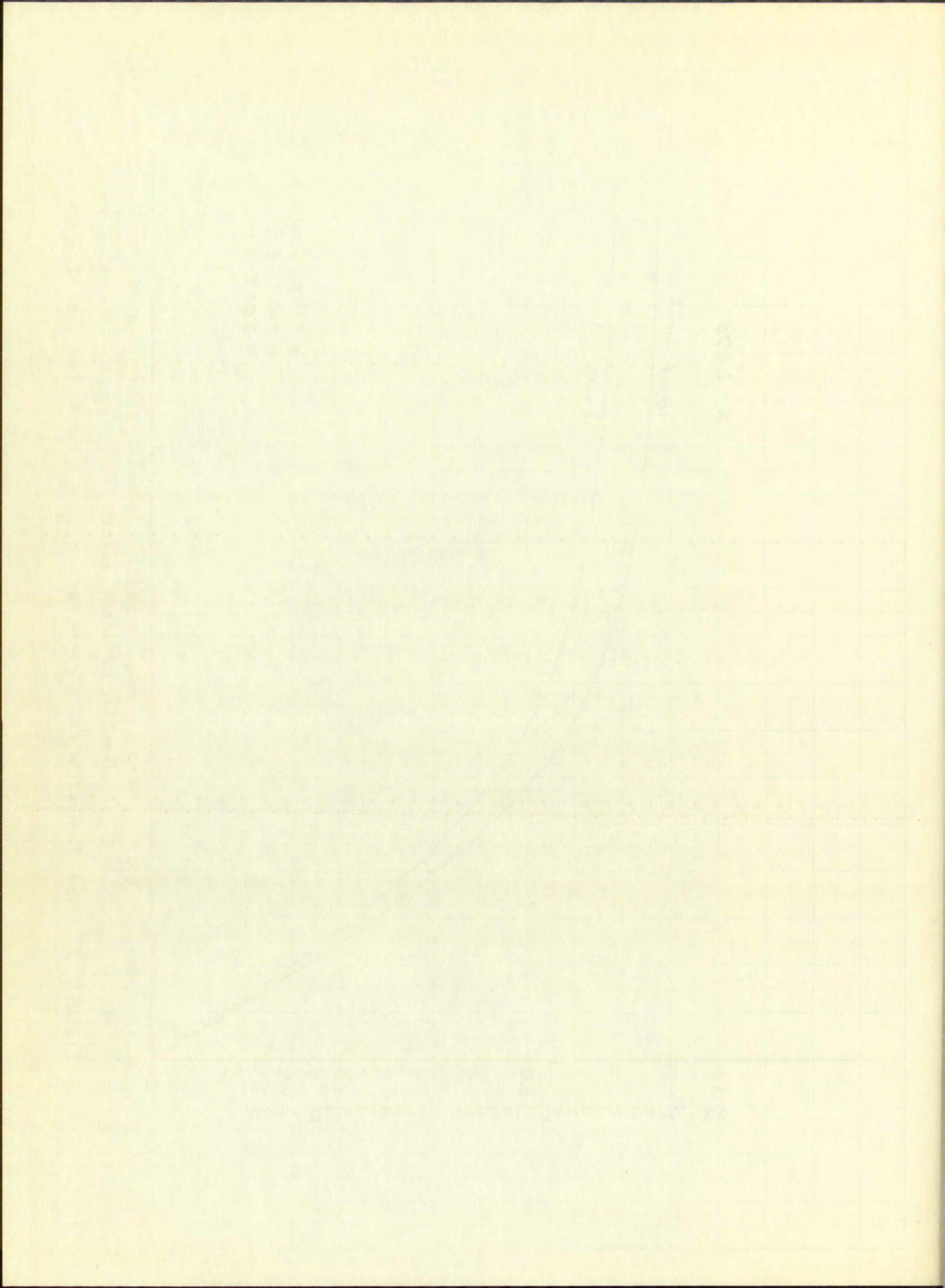


FIGURE 7 Non-Dimensional Temperature vs. Time for Tests 4, 5, 6



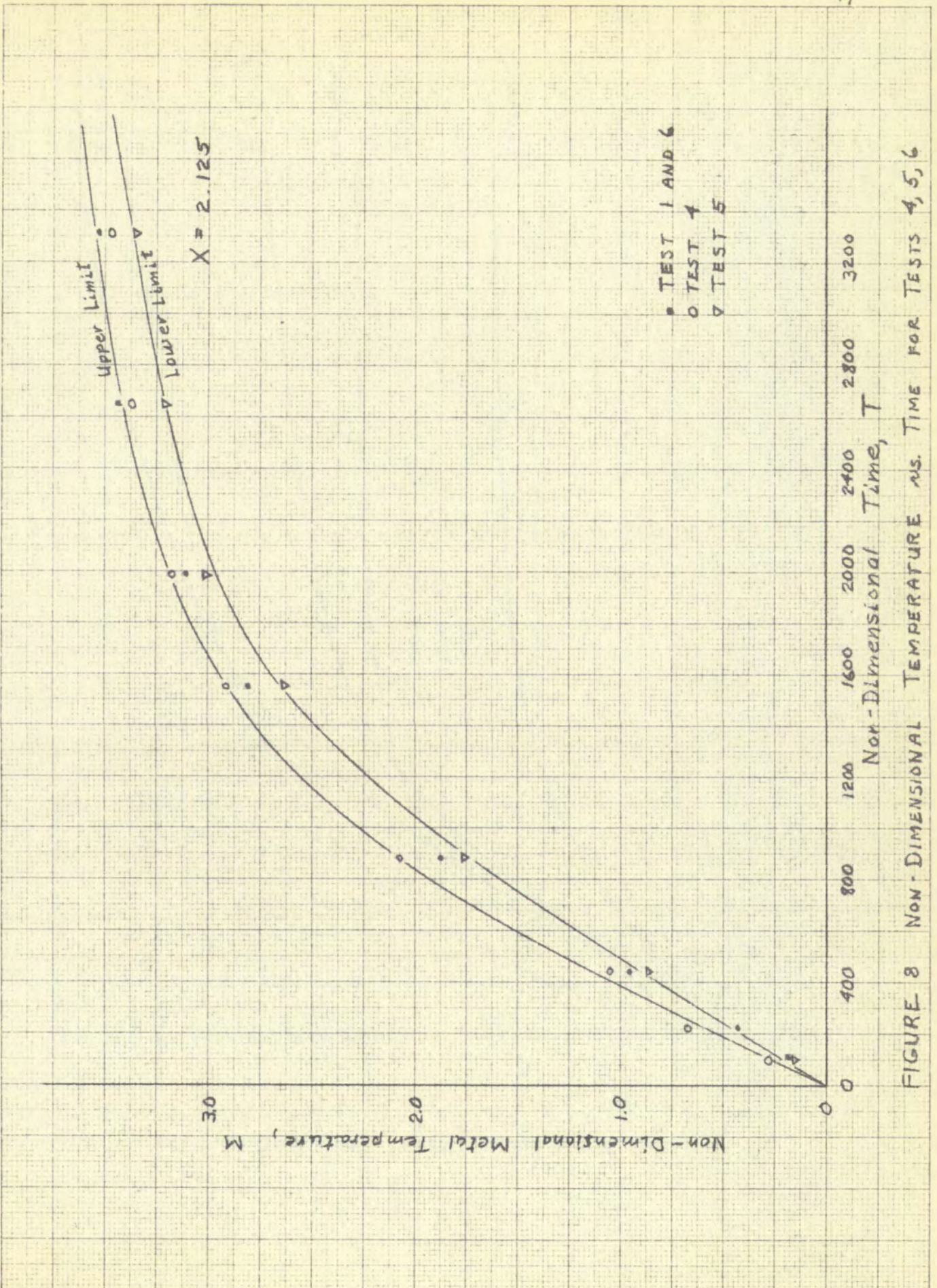
Upper Limit
 Lower Limit
 $X = 2.125$

• TEST 1 AND 6
 ○ TEST 4
 ▽ TEST 5

Non-Dimensional Metal Temperature, M

Non-Dimensional Time, T

FIGURE 8 NON-DIMENSIONAL TEMPERATURE VS. TIME FOR TESTS 4, 5, 6



UNIVERSITY OF CALIFORNIA
DEPARTMENT OF CHEMISTRY
LABORATORY OF POLYMER CHEMISTRY
BERKELEY, CALIF. 94720

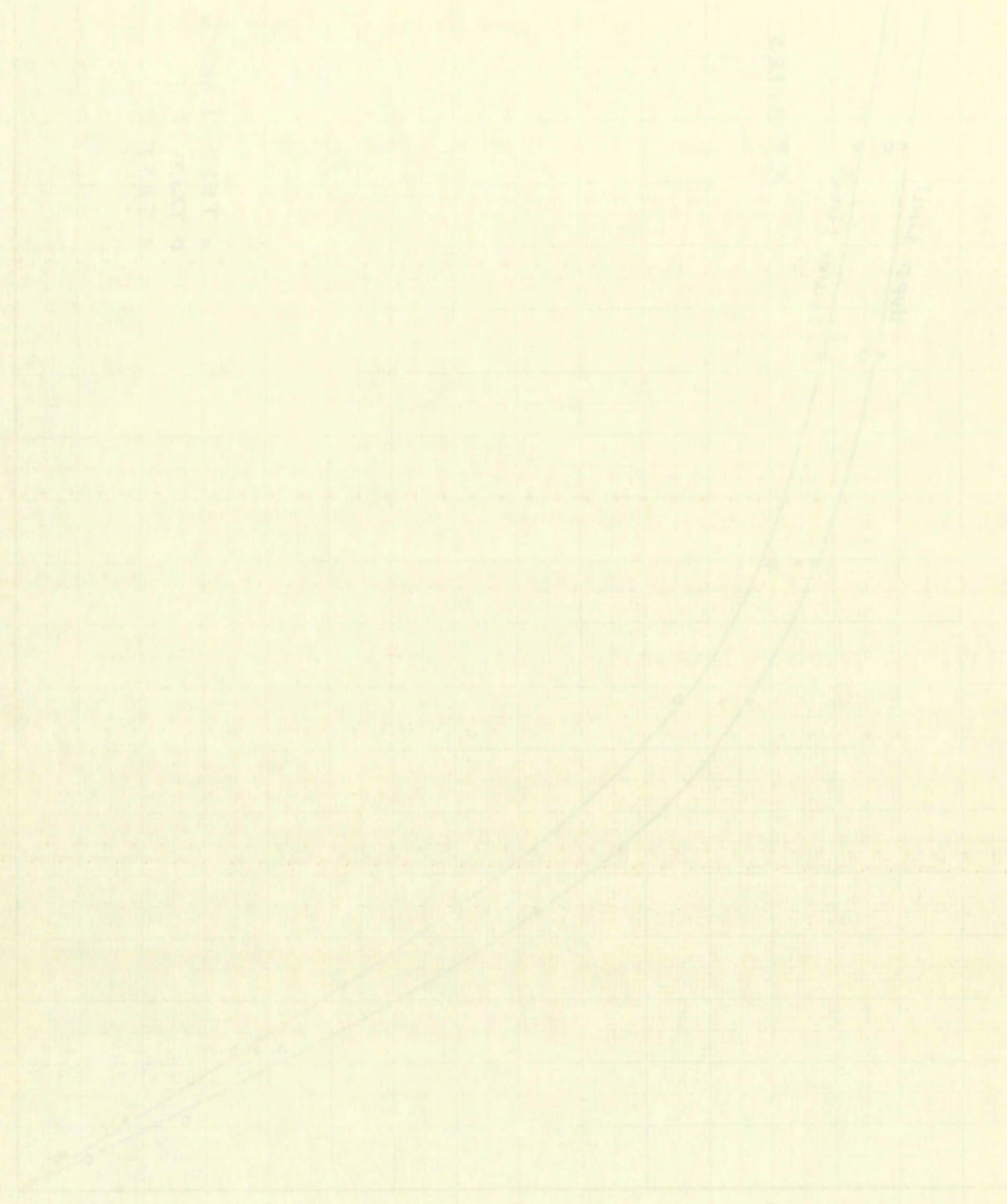


Fig. 1. Molecular weight vs. time for polymerization at 100°C and 120°C.

DATE: 10/10/50

LABORATORY OF POLYMER CHEMISTRY
UNIVERSITY OF CALIFORNIA
BERKELEY, CALIF.

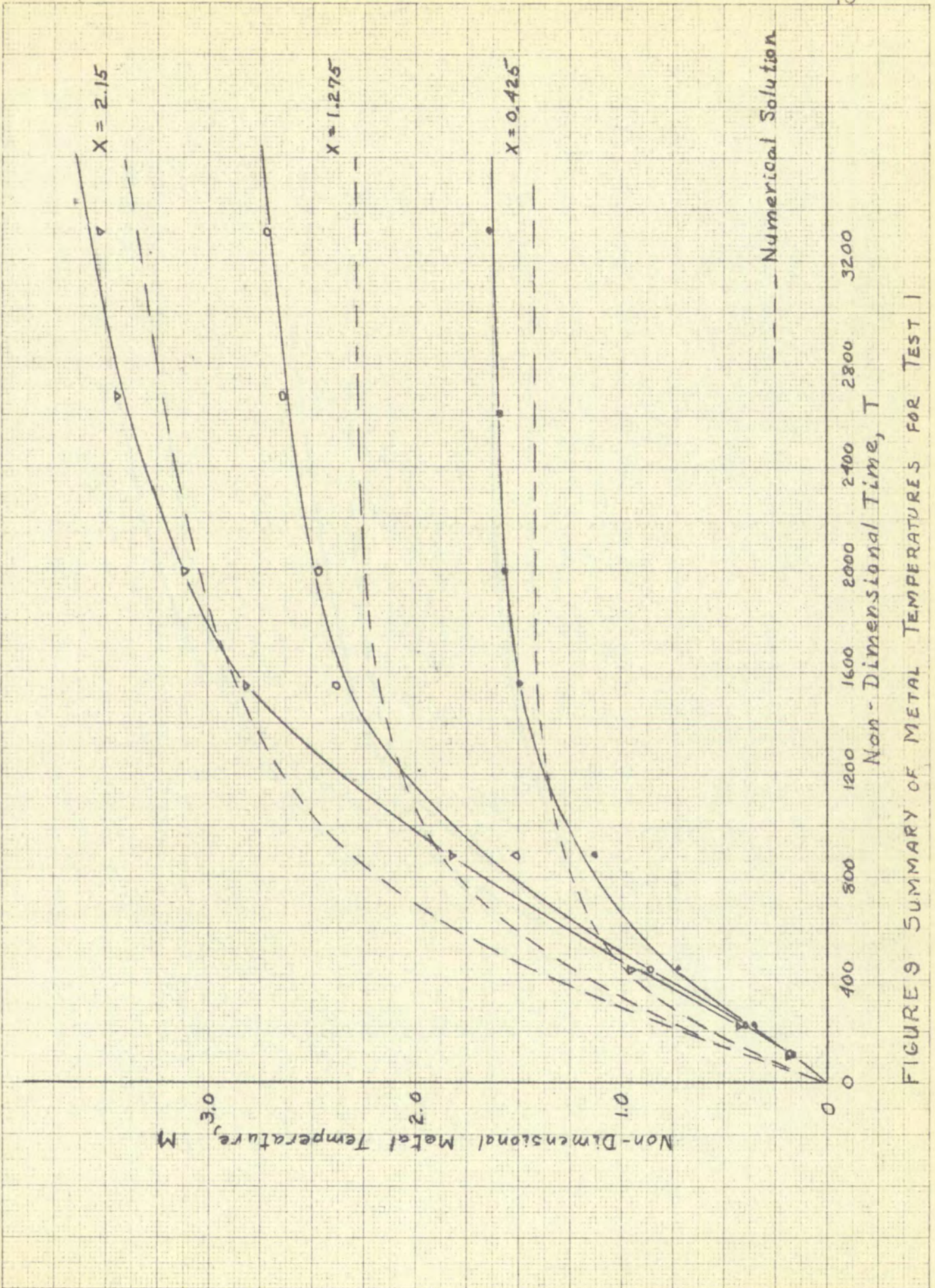


FIGURE 9 SUMMARY OF METAL TEMPERATURES FOR TEST 1

STRENGTH OF CONCRETE IN TENSION

STRENGTH OF CONCRETE IN TENSION

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STRENGTH OF CONCRETE IN TENSION

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STRENGTH OF CONCRETE IN TENSION

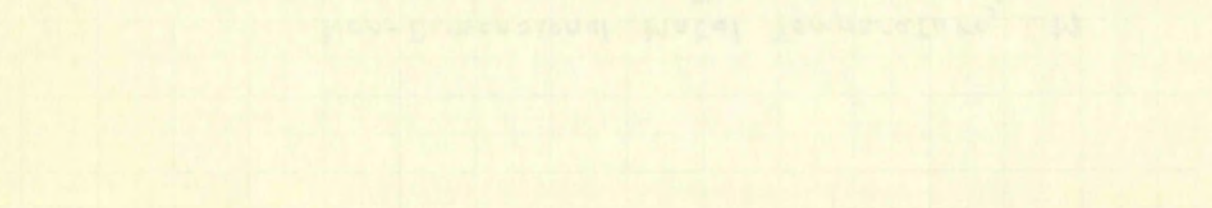
STRENGTH OF CONCRETE IN TENSION

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STRENGTH OF CONCRETE IN TENSION



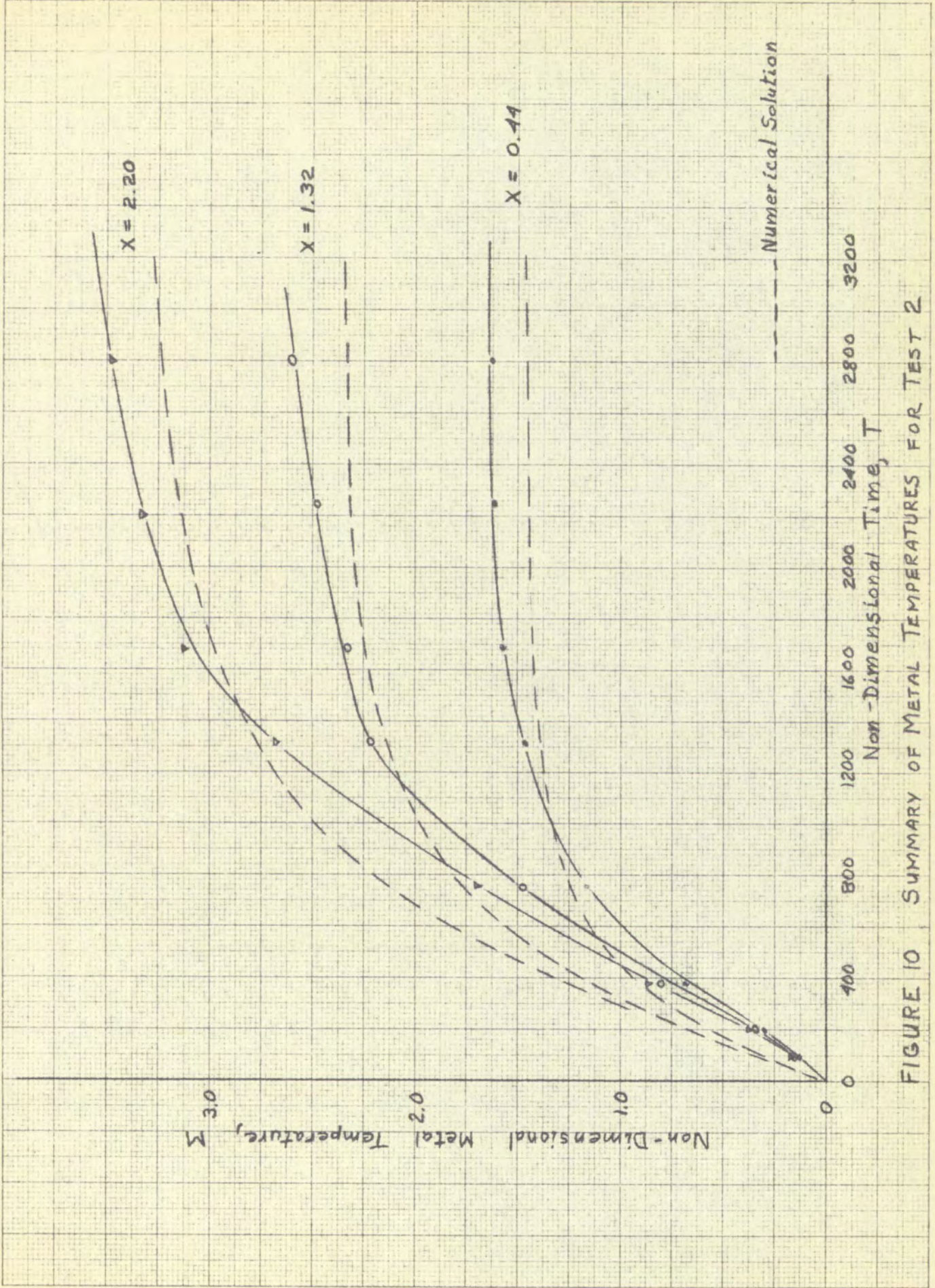
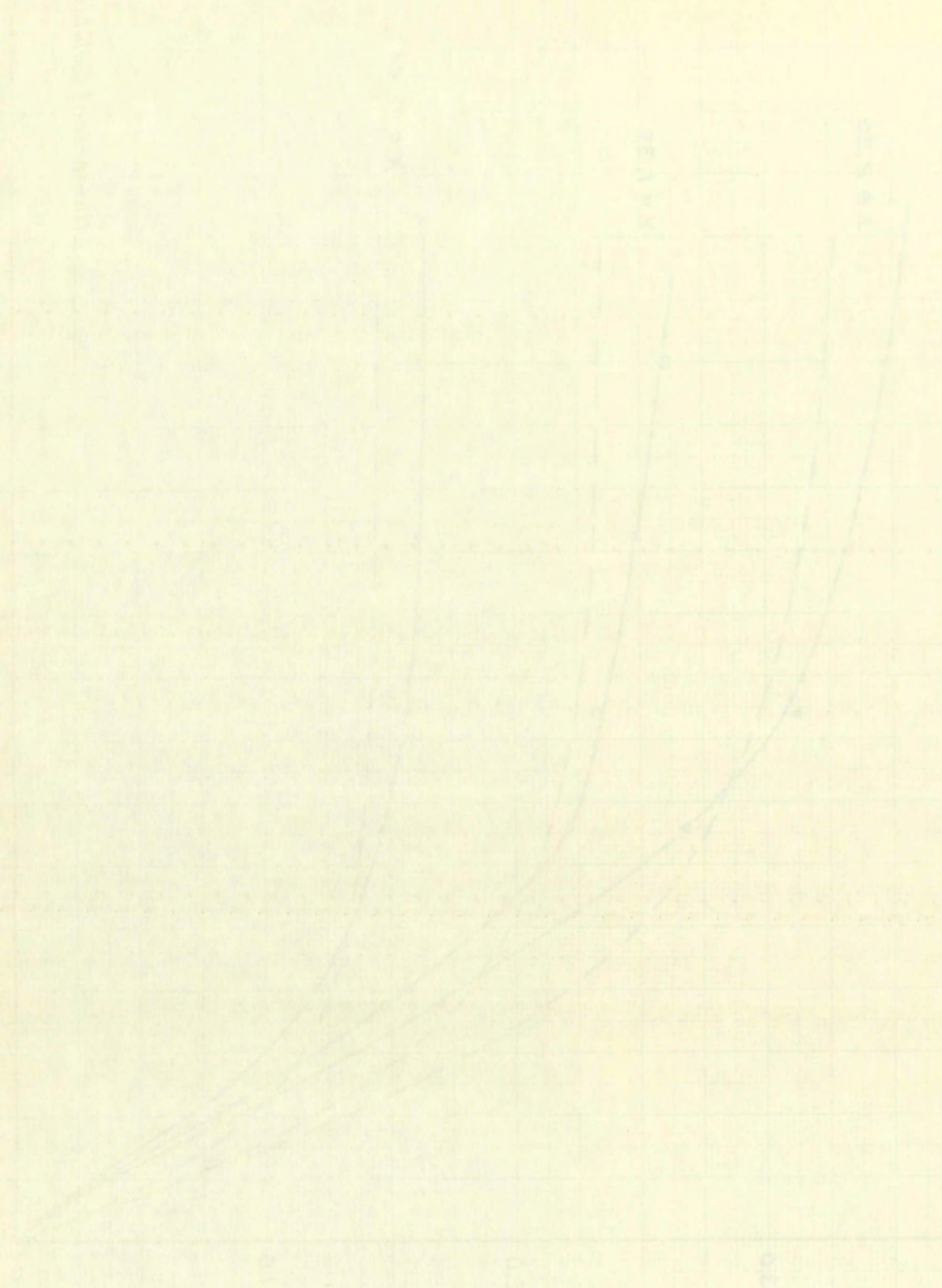


FIGURE 10 SUMMARY OF METAL TEMPERATURES FOR TEST 2



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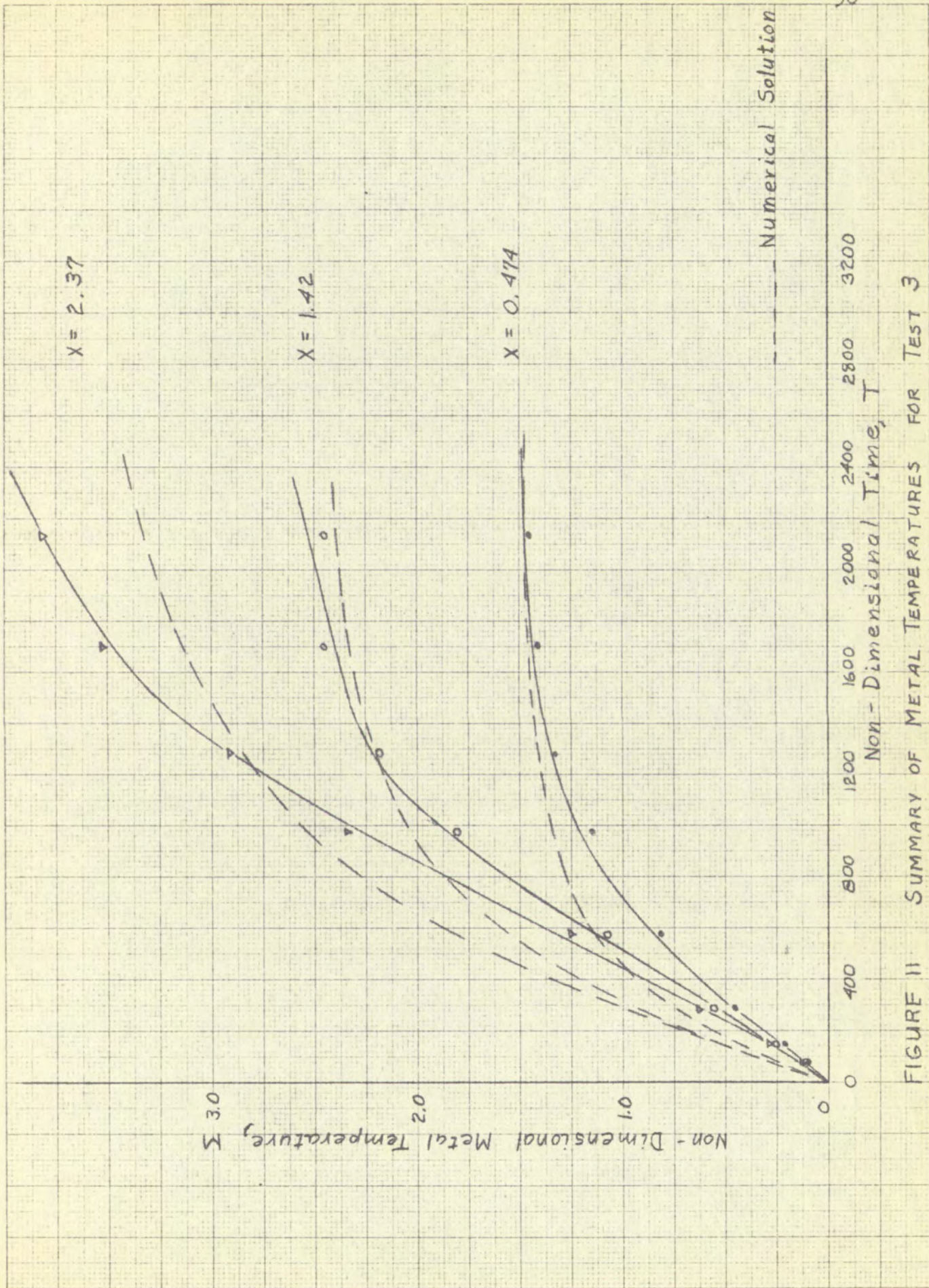
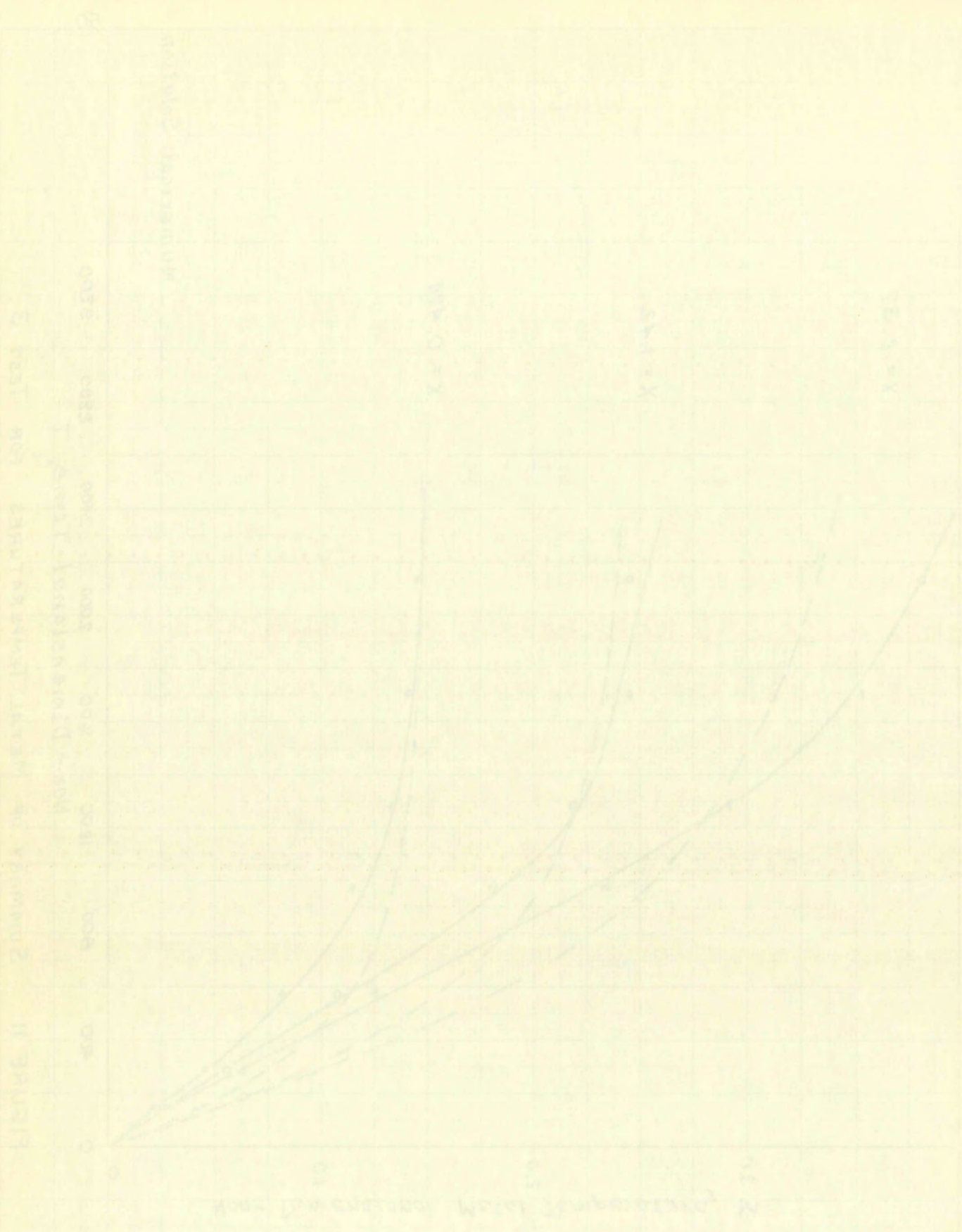


FIGURE II SUMMARY OF METAL TEMPERATURES FOR TEST 3



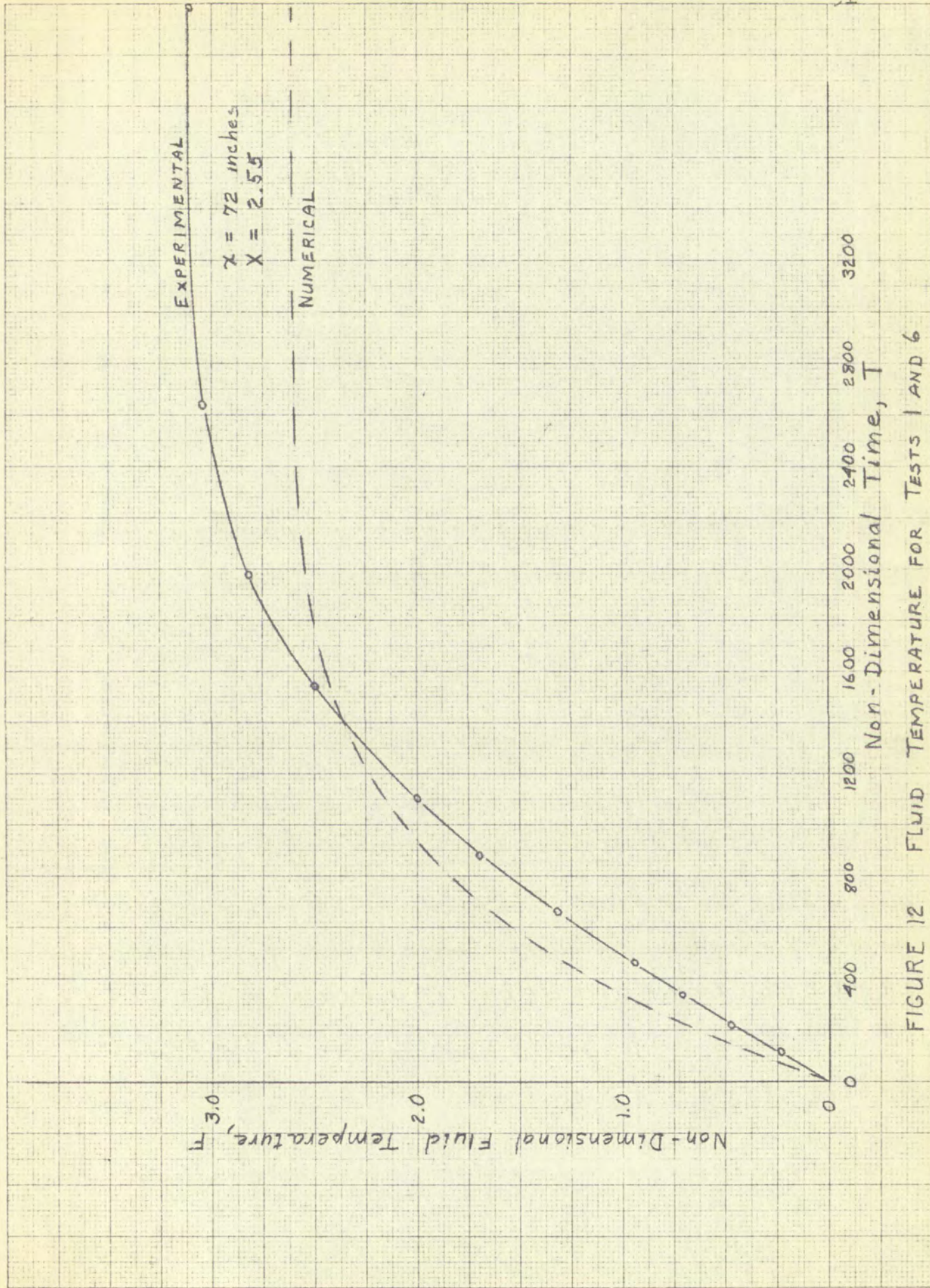
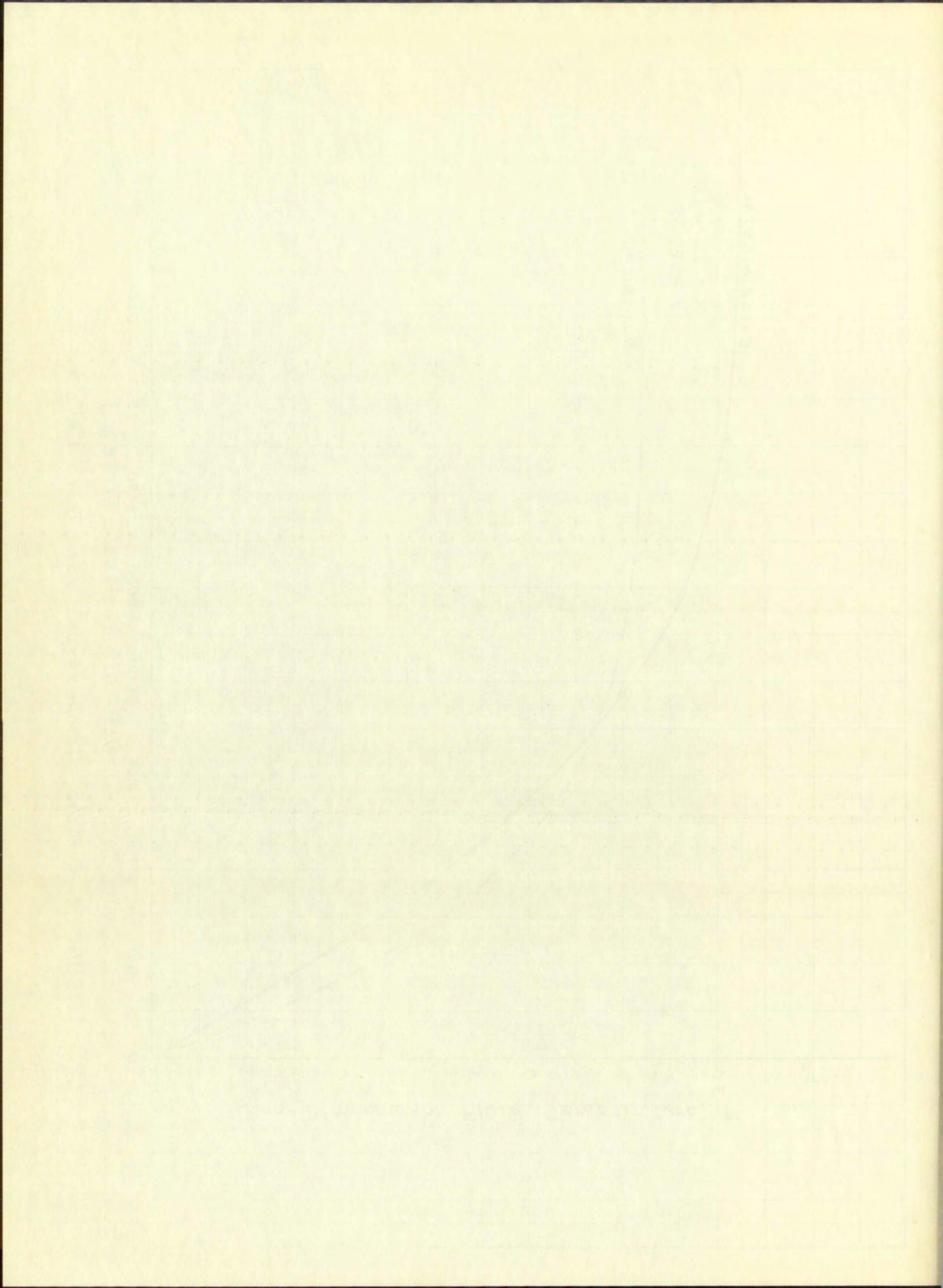


FIGURE 12 FLUID TEMPERATURE FOR TESTS 1 AND 6



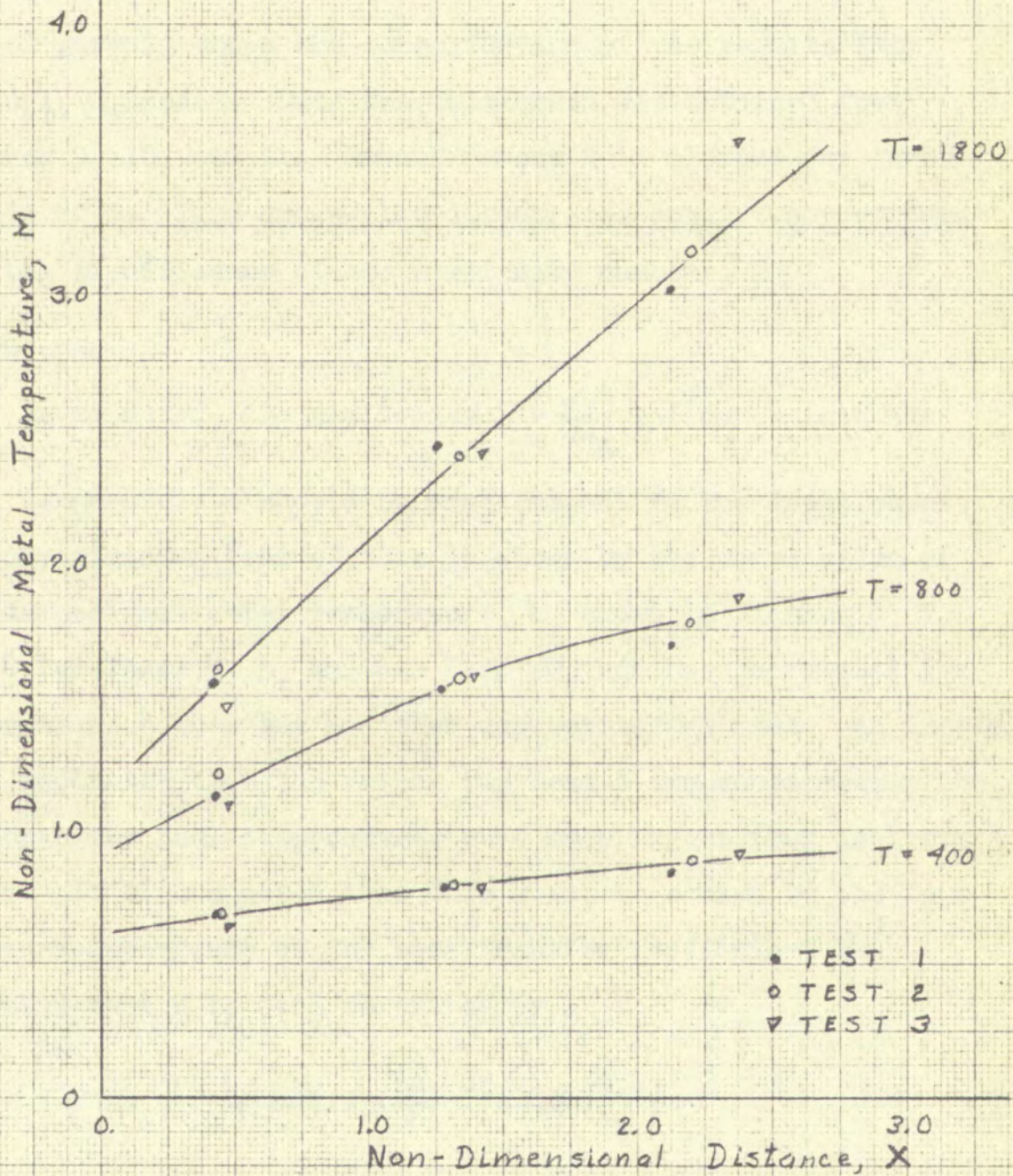


FIGURE 13 CROSS-PLOT OF RESULTS
FROM TESTS 1, 2, 3

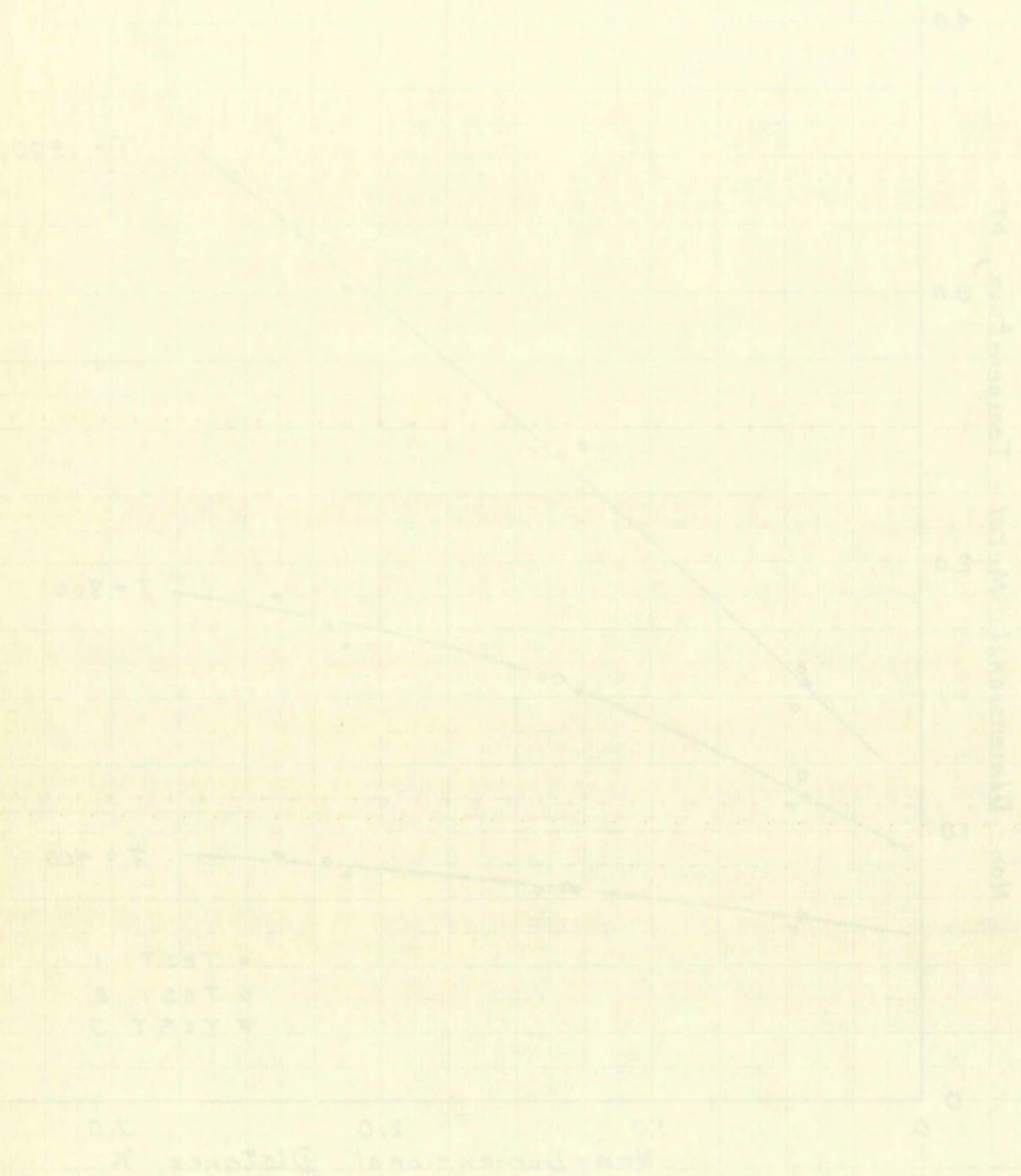


FIGURE 13. Comparison of Results from Tests 1, 2, & 3.

for $X = 2.55$ which corresponds to $x = 72$ inches for those tests. The theoretical curve for the same X is plotted with the experimental results.

Figure 13 shows the consolidation of the results from Tests 1, 2, and 3. Data for this graph was obtained from Figures 9, 10, and 11. Here M versus X is plotted for three values of T . This cross-plot illustrates consistency of data and the significance of the similarity numbers.

DISCUSSION

As expected, the similarity $M = \frac{mhp}{\Delta U_g}$ indicates that the real temperature m should be proportional to the input power. The experimental test of this is shown by the correlation of non-dimensional metal temperature, M , shown in Figures 6, 7, and 8 for Tests 4, 5, and 6. Here ΔU_g has been corrected for external heat loss but no other correction was made. An inspection indicates that the values for Test 5 are consistently low although the real temperatures were much higher than in Tests 4 and 6. It is suspected that this might be caused by the temperature dependence of the heat transfer coefficient h . Assuming that h is well described by

$$\frac{hD}{k} = 0.23 \left(\frac{DG}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4}, \quad 29$$

the temperature dependence may be attributed to k and μ by

$$h \propto \frac{(k)^{0.6}}{(\mu)^{0.4}},$$

The experimental error for the same X is plotted with the experimental results.

Figure 13 shows the consolidation of the results from Table I, II, and III. The data points obtained from Figures 10, 11, and 12. Here the same X is plotted for each value of T. This weighted linearized consistency of data and the alignment of the straight lines.

It is expected that the straight line $N = \frac{1}{2} U$ indicates that the

total temperature N should be proportional to the input power. The experimental test of this is shown by the correlation of non-dimensional total temperature N , shown in Figures 5, 6, and 7 for Tests 4, 5, and 6. Here U has been corrected for external heat loss but no other correction was made. It is noted that the values for Test 5 are consistently low although the total temperatures were much higher than in Tests 4 and 6. It is suspected that this might be caused by the low relative dependence of the heat transfer coefficient h , assuming that h is well described by

$$\frac{h}{k} = 0.22 \left(\frac{U}{k} \right)^{0.8} \left(\frac{D}{L} \right)^{-0.1}$$

The temperature dependence may be attributed to h and U .

$$h = \frac{0.22}{0.1} U^{0.8} D^{-0.1}$$

and

$$\frac{h_4}{h_5} = \left(\frac{\mu_5}{\mu_4}\right)^{0.4} \left(\frac{k_4}{k_5}\right)^{0.6} \quad . \quad 32$$

The variables are evaluated as follows:

$$\mu_4 = 0.021 \text{ at } 200^\circ\text{F}$$

$$\mu_5 = 0.029 \text{ at } 600^\circ\text{F}$$

$$k_4 = 0.0184 \text{ at } 200^\circ\text{F}$$

$$k_5 = 0.0260 \text{ at } 600^\circ\text{F}$$

Substitutions of these values in equation 32 gives

$$h_4 = h_5(1.38)^{0.4}(0.707)^{0.6}$$

or

$$h_4 = 0.925 h_5.$$

The dimensionless M is directly proportional to h. Therefore, a deviation of approximately 7-1/2% between the curves for Test 4 and Test 5 is explained by the temperature dependence of h. This is about the magnitude of the deviation in Figures 6, 7, and 8. Taking the above into account, the agreement between tests is sufficient to justify the method by which ΔU_g was corrected for external heat loss.

The principal results of the investigation are illustrated by Figures 9, 10, 11, and 12. The departure of the theoretical curves from the experimental curves is characterized by an

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2} + \dots + \frac{1}{\sigma_n^2}$$

The variables are defined as follows:

$$M_1 = 0.025 \text{ at } 200^\circ\text{C}$$

$$M_2 = 0.035 \text{ at } 250^\circ\text{C}$$

$$M_3 = 0.045 \text{ at } 300^\circ\text{C}$$

$$M_4 = 0.055 \text{ at } 350^\circ\text{C}$$

Substitutions of these values in equation (1) gives

$$\frac{1}{\sigma^2} = \frac{1}{0.025^2} + \frac{1}{0.035^2} + \frac{1}{0.045^2} + \frac{1}{0.055^2}$$

or

$$\frac{1}{\sigma^2} = 15.87 \text{ hr.}^{-2}$$

The dimensionless M is directly proportional to the time t .

A deviation of approximately 10% in the values of M will result in a deviation of approximately 20% in the values of t .

Test 1 and Test 2 are repeated by the same procedure.

of t . This is about the magnitude of the deviation in t .

6, 7, and 8. Tables are also included in the appendix.

These tests are sufficient to determine the values of M .

was corrected for external test errors.

The primary results of the present study are summarized in

by Figures 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100.

curves from the experimental data in order to compare the

initial overstatement and a final understatement of temperatures. The cross-over occurs at a time corresponding to about half of the time required to reach equilibrium which is achieved in the same time for both. Considering that the numerical computation assumed fluid properties at a median temperature of 300°F, the comparison is good.

The consistency of data is partially shown by the agreement among tests in Figures 6, 7, and 8. A comprehensive demonstration of consistency is presented by the cross-plot in Figure 13. In non-dimensional terms, Figure 13 indicates that the similarity theory, experimental data and calculations are probably valid since the results are consistent and are comparable to the numerical solution.

initial overstatement and a final understatement of temperature. The maximum occurs at a time corresponding to about half of the time required to reach equilibrium when it is reached in the same time for both. Considering that the numerical calculation used in this report is at a higher temperature of 300°F, the error is likely to be small. The consistency of data is partially shown by the agreement among tests in Figures 6, 7, and 8. A comprehensive description of consistency is presented in the appendix in Figure 13. In non-dimensional terms, Figure 13 indicates that the similarity theory, experimental data and calculations are probably valid since the results are consistent and are comparable to the numerical solution.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn from this investigation:

1. Transient performance of a heat exchanger with an internal source can be correlated by use of the similarity numbers $M = \frac{mhp}{\Delta U_g}$, $F = \frac{fhp}{\Delta U_g}$, $X = \frac{fhp}{\Delta U_g}$,

$$T = \frac{thp}{c_f \delta_f A_f}, \text{ and } a = \frac{c_m \delta_m A_m}{c_f \delta_f A_f}.$$

2. The use of an average temperature for fluid properties gives a numerical solution which overstates initial temperatures and understates final temperatures for the conditions of this investigation.
3. The assumption of constant fluid properties may lead to significant error when temperature changes are 400°F or greater.
4. The analytical solution must be further simplified before it becomes useful as a design tool.

An analytical solution of this problem for design application has still not been achieved and it is recommended that further efforts be directed toward the simplification of the results obtained in this investigation. The approach to this problem might be the division of the problem into the special cases of small "a" and large "a," small "X" and large "X," etc.

CHAPTER 3

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions can be drawn from this investigation:

1. Theoretical performance of a heat exchanger with an

internal heater can be correlated by use of the

$$\frac{NTU}{Z} = \frac{1}{2} \left(\frac{1}{1 + \frac{1}{2} \frac{Z}{NTU}} \right) \left(\frac{1}{1 + \frac{1}{2} \frac{Z}{NTU}} \right)$$

$$Z = \frac{2 \ln \left(\frac{1 + \frac{1}{2} \frac{Z}{NTU}}{1 - \frac{1}{2} \frac{Z}{NTU}} \right)}{\frac{1}{2} \frac{Z}{NTU}}$$

2. The use of an average temperature for fluid properties

gives a numerical solution which overstates

initial temperatures and understates final temperatures

for the conditions of this investigation.

3. The assumption of constant fluid properties may

lead to significant error when temperature changes

are 400° or greater.

4. The analytical solution may be further simplified

before it becomes useful as a design tool.

An analytical solution of this problem for design appli-

cation has still not been achieved and it is recommended that

further efforts be directed toward the simplification of the

analysis obtained in this investigation. The approach to this

problem might be the division of the problem into the special

cases of small "a" and large "a", small "X" and large "X", etc.

It should be borne in mind, however, that the analytical solution may not represent the physical problem very closely unless temperature dependent effects are accounted for. This suggests that a more sophisticated numerical method might be more useful, but a digital computer should be used to reduce the labor.

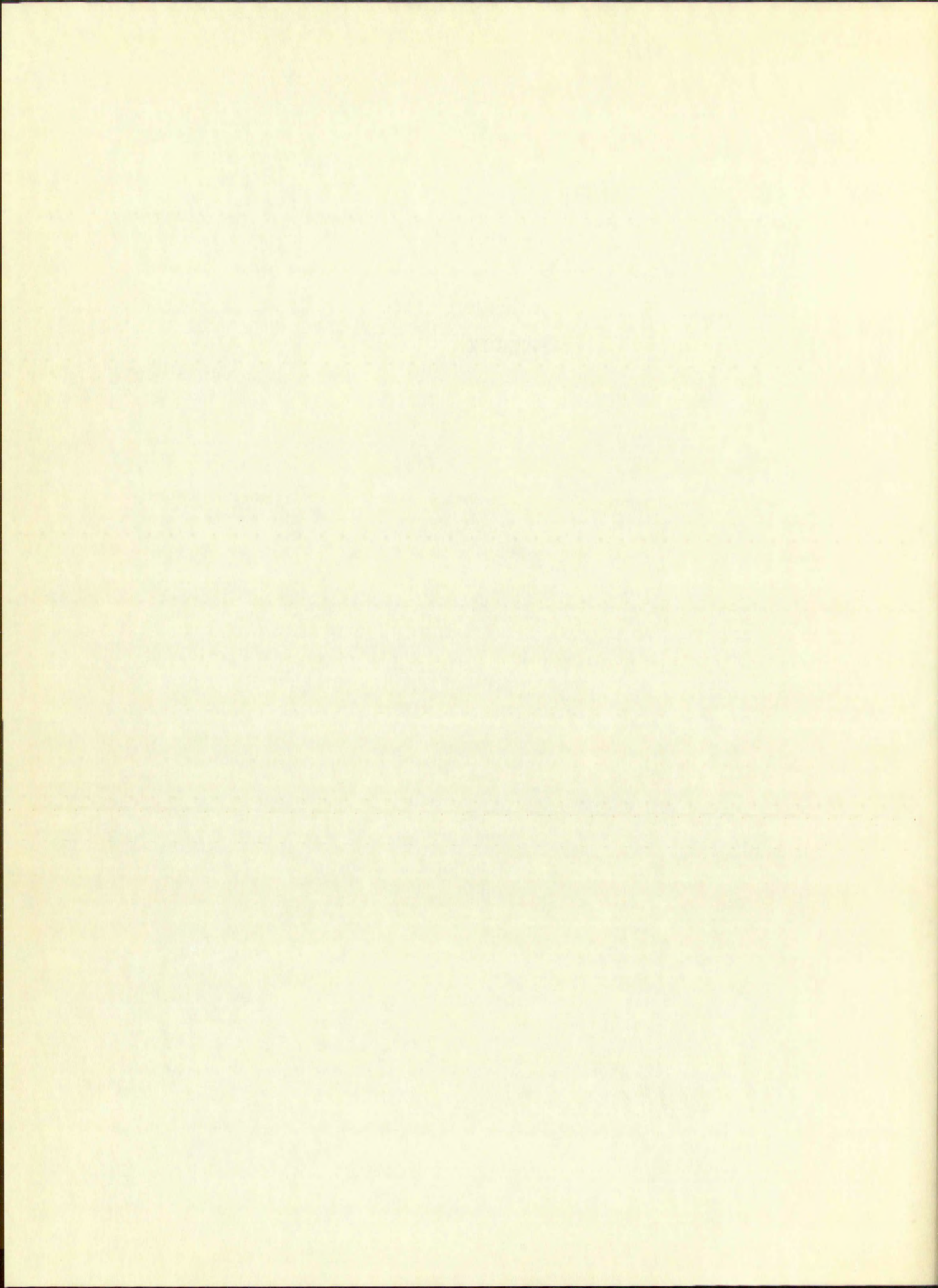
Additional work on the thermal analog discussed here might be rewarding. The similarity numbers derived appears to be quite significant, and it is believed that the analog approach should give good results which would be valuable in design studies. Further analog results would also be of assistance in deriving, as well as verifying, new theoretical approaches.

Some refinements are recommended in the present apparatus. The starting point should be the development of a better method for installing thermocouples on the tube. A low-heat-capacity insulation around the heated tube would reduce external heat losses and improve the precision of the results. The possibility of measuring bulk coolant temperature at several points might also be investigated. A more stable power source and a larger air supply tank might contribute to slight improvements in data.

It should be pointed out that the...
additional work of the...
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APPENDIX

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COMPARISON OF THERMOMETER IN SPECIAL WELL
AND TUBE THERMOCOUPLES

Test A - Comparison with iron-constantan thermocouple at $x = 36$
inches. Date: 14 June 1958

Thermometer in well °F	Chromel-Alumel TC in well °F	Iron-Constantan TC on tube °F
equilibrium		
598	592	592
605	600	607
tube cooling		
	535	520
readings taken in succession	500	482
	230	225
	220	218
	213	210
	207	207

THE UNIVERSITY OF CHICAGO
DEPARTMENT OF CHEMISTRY
1950

RESEARCH REPORT NO. 100
BY J. H. GOLDSTEIN AND R. F. SCHWENKER

THE EFFECT OF TEMPERATURE ON THE
RATE OF REACTION OF
HYDROGEN PEROXIDE WITH
SODIUM HYDROGEN SULFATE

RECEIVED BY THE LIBRARY OF THE
UNIVERSITY OF CHICAGO
ON APRIL 10, 1951

CHICAGO, ILLINOIS
1951

PRINTED IN THE UNITED STATES OF AMERICA

TABLE 1 (Continued)

COMPARISON OF THERMOMETER IN SPECIAL WELL
AND TUBE THERMOCOUPLES

Test B - Comparison with chromel-alumel thermocouple at x = 60 inches.

Date: 17 June 1958

Thermometer in well °F	Chromel-Alumel TC in well °F	Chromel-Alumel TC on tube °F
equilibrium		
640	642	662
640	642	662
tube cooling		
	337	322
readings taken in succession	303	294
	288	283
	278	275
	270	270
	262	259

1. The first part of the document is a letter from the President of the United States to the Congress, dated September 17, 1787. It is a very important document, as it is the first time that the President has addressed the Congress. The letter is a very long and detailed one, and it covers a wide range of topics. It is a very important document, as it is the first time that the President has addressed the Congress. The letter is a very long and detailed one, and it covers a wide range of topics.

2. The second part of the document is a letter from the President of the United States to the Congress, dated September 17, 1787. It is a very important document, as it is the first time that the President has addressed the Congress. The letter is a very long and detailed one, and it covers a wide range of topics. It is a very important document, as it is the first time that the President has addressed the Congress. The letter is a very long and detailed one, and it covers a wide range of topics.

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TABLE 2
 FACTOR K FOR CONVERTING OSCILLOGRAPH
 TRACE DEFLECTION TO TEMPERATURE

Circuit	Factor K = °F/inch	Probable Error, °F (Assuming ± 1/64" reading accuracy)
Galv. No. 2 - T.C. at 1/2" or 6"	657	± 10
Galv. No. 3 - T.C. at 12"	577	± 9
Galv. No. 4 - T.C. at 36"	593	± 9
Galv. No. 5 - T.C. at 60"	1130	± 18
Galv. No. 6 - T.C. at 66" or 71-1/2"	1040	± 16
Galv. No. 10 with amplifier - Fluid Temp. Probe	235	± 4

TABLE 2 - THE COMPANY'S FINANCIAL STATEMENTS

ASSETS

Assets	1967	1968	1969
Current assets	100	100	100
Fixed assets	100	100	100
Total assets	200	200	200
Liabilities	100	100	100
Equity	100	100	100
Total liabilities and equity	200	200	200

TABLE 3

EXTERNAL HEAT LOSS MEASUREMENT

Date: 17 June 1958Test No. 1

Input Power, watts/inch	Metal Temp. °F at x = 1/2"	Metal Temp. °F at x = 12"	Metal Temp. °F at x = 36"	Metal Temp. °F at x = 60"	Metal Temp. °F at x = 72-1/2"
0.69	56	88	92	90	43
1.47	118	175	178	178	91
3.10	189	287	303	305	157
4.89	272	404	427	440	228
7.59	357	523	550	575	317
10.0	430	625	657	708	398

Date: 21 June 1958Test No. 2

Input Power, watts/inch	Metal Temp. °F at x = 6"	Metal Temp. °F at x = 12"	Metal Temp. °F at x = 36"	Metal Temp. °F at x = 60"	Metal Temp. °F at x = 66"
1.34	157	153	160	156	150
2.28	233	230	240	154	236
4.08	365	355	369	390	390
7.03	528	506	530	608	590

EXHIBIT 1015 FORD MOTOR CO.

TABLE 1

Sheet No. 1 of 1

Sheet No. 1 of 1

TABLE 4.

RAW DATA - METAL TEMPERATURES

Date: 21 June 1958
 Test No. 1

Flow Rate: 0.865 #/min
 Power Input: 30.9 watts/in
 Inlet Air Temp.: 82°F; Reference Junction (Room Temp.): 85°F

Time sec	Temp. (°F) at x = 6"			Temp. (°F) at x = 12"			Temp. (°F) at x = 36"			Temp. (°F) at x = 60"			Temp. (°F) at x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	55	51	53	30	38	34	32	45	39	42	52	47	40	50	45
10	95	102	99	80	75	78	100	105	103	100	108	104	95	105	100
15	126	137	132	121	122	122	140	157	149	140	148	144	145	150	148
20	181	176	179	167	164	166	198	210	204	200	205	203	205	220	213
30	230	223	227	227	223	225	283	284	284	282	282	282	310	295	303
40	268	255	262	275	267	271	345	340	343	358	355	357	390	385	388
50	287	272	280	305	293	299	390	392	391	420	410	415	455	440	448
70	312	297	305	340	322	331	456	444	450	510	490	500	540	520	530
90	325	308	317	355	335	345	482	468	475	575	545	560	600	575	588
120	317	321	319	350	348	349	492	488	490	595	580	588	615	600	608
150	327	315	321	360	342	351	498	490	494	608	580	594	620	595	608
measured	306	335	321	334	329	332	472	473	473	598	585	592	610	588	599

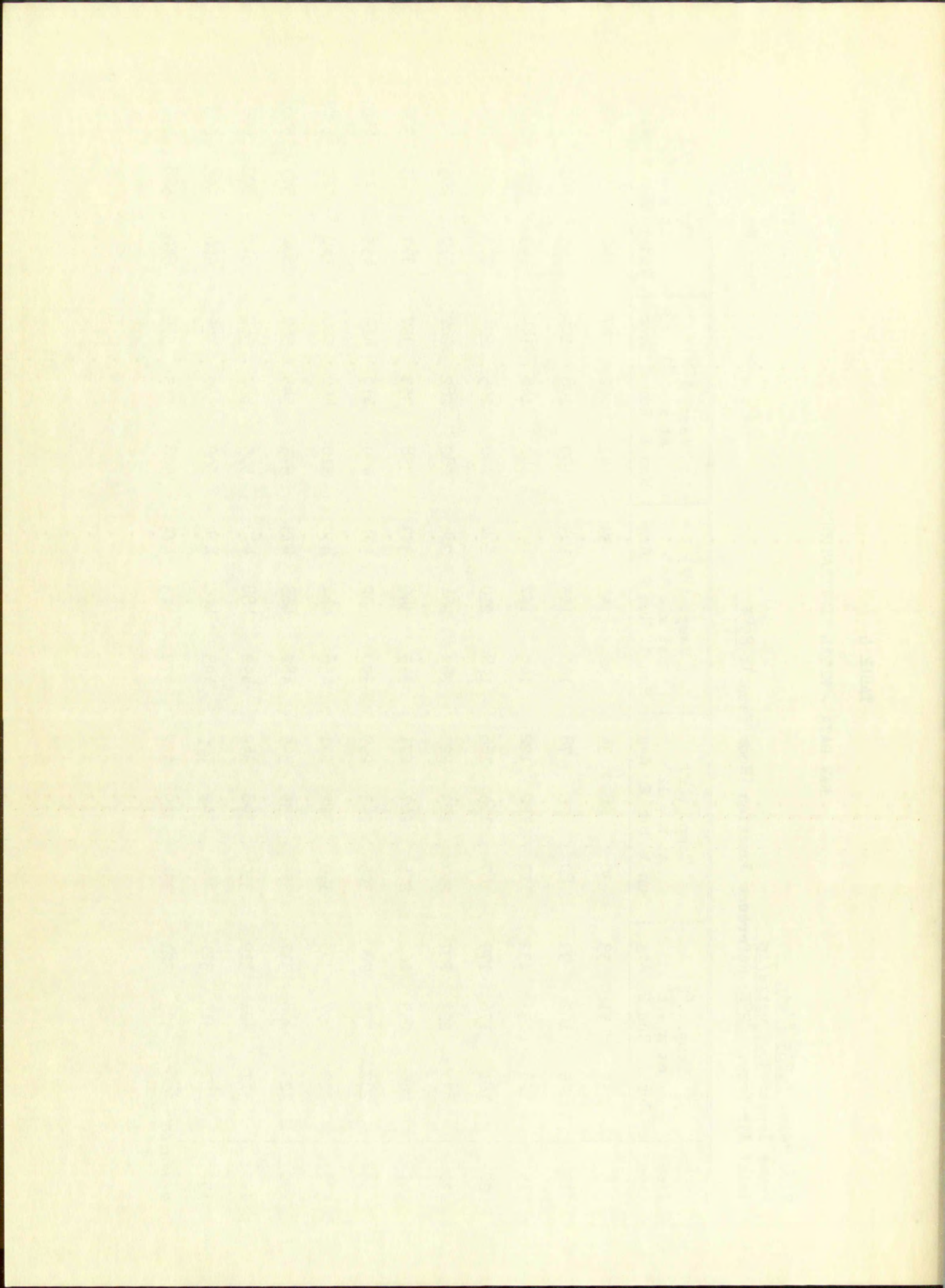


TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 21 June 1958Test No. 2Flow Rate: 0.708 #/minPower Input: 30.9 watts/inInlet Air Temp.: 85°F; Reference Junction (Room Temp.): 86°F

Time sec	Metal Temp. (°F) at x = 6"			Metal Temp. (°F) at x = 12"			Metal Temp. (°F) at x = 36"			Metal Temp. (°F) at x = 60"			Metal Temp. (°F) at x = 66"		
	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG
5	-	40	-	38	28	32	44	40	42	50	45	48	55	40	48
10	-	82	-	85	78	81	88	97	93	100	90	95	105	95	100
15	-	127	-	135	130	133	148	160	154	165	155	160	170	160	165
20	-	181	-	177	165	171	227	210	219	220	205	213	230	215	223
30	-	243	-	242	237	240	313	298	306	310	295	303	335	305	320
40	-	275	-	288	282	285	387	375	381	395	370	383	415	390	403
50	-	296	-	326	306	317	438	422	430	475	445	460	480	450	465
70	-	323	-	362	346	354	502	480	491	578	540	559	575	540	558
90	-	345	-	378	358	368	532	520	526	630	605	618	620	590	605
120	-	332	-	378	380	279	532	542	540	670	695	683	670	630	650
150	-	320	-	382	372	279	552	548	550	675	725	700	665	640	653
measured	347	351	349	349	389	269	531	543	537	667	685	676	667	685	676

Year	1950	1951	1952	1953	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964	1965	1966	1967	1968	1969	1970
1950	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1951	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1952	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1953	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1954	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1955	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1956	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1957	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1958	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1959	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1960	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1961	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1962	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1963	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1964	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1965	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1966	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1967	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1968	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1969	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
1970	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100

UNITED STATES GOVERNMENT
 NATIONAL BUREAU OF ECONOMIC RESEARCH
 FEDERAL RESERVE BOARD
 WASHINGTON, D. C. 20540
 1970

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 21 June 1958

Test No. 3

Flow Rate: 0.50 #/min

Power Input: 30.9 watts/in

Inlet Air Temp.: 87°F; Reference Junction (Room Temp.): 86°F

Time Sec	Metal Temp. (°F) x = 6"		Metal Temp. (°F) x = 12"		Metal Temp. (°F) x = 36"		Metal Temp. (°F) x = 60"		Metal Temp. (°F) x = 66"	
	Run 1	Run 2	Run 1	Run 2	Run 1	Run 2	Run 1	Run 2	Run 1	Run 2
5	35	40	37	33	48	45	40	50	45	45
10	68	82	74	80	100	102	85	100	85	105
15	102	120	122	124	162	168	145	160	155	150
20	125	153	160	169	208	218	190	212	200	215
30	183	198	241	234	318	304	295	300	305	305
40	223	235	302	285	405	382	390	380	395	385
50	246	263	341	323	470	445	485	465	485	470
70	274	302	400	383	553	537	620	615	595	600
90	294	321	427	412	605	592	710	700	685	675
120	300	335	445	438	643	627	775	735	735	740
150	308	304	448	438	663	640	825	770	775	745
measured	445	450	477	478	647	655	821	830	802	795
				478	478	651	826	826	795	799

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Flow Rate: 0.865 #/minPower Input: 13.9 watts/inInlet Air Temp.: 85°F; Reference Junction (Room Temp.): 88°FDate: 23 June 1958Test No. 4

Time sec	Metal Temp. (°F) x = 6"			Metal Temp. (°F) x = 12"			Metal Temp. (°F) x = 36"			Metal Temp. (°F) x = 60"			Metal Temp. (°F) x = 66"		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	20	15	18	24	20	22	20	23	22	32	30	31	25	30	28
10	42	25	31	42	35	39	45	46	46	52	45	49	53	55	54
15	56	35	47	61	56	59	71	73	72	73	60	67	80	82	81
20	70	47	59	82	77	80	96	96	96	93	92	93	110	110	110
30	90	56	73	112	102	107	137	133	135	142	123	133	158	147	153
40	106	67	87	130	123	127	165	165	165	168	159	164	197	200	199
50	111	68	90	142	133	138	188	190	189	185	182	184	232	220	226
70	116	75	96	155	138	147	213	206	210	220	208	214	268	258	253
90	122	80	101	156	145	151	218	216	217	235	223	229	278	280	279
120	122	80	101	157	146	153	230	228	229	248	242	245	302	282	292
150	125	-	-	165	-	-	234	-	-	260	-	-	312	-	-
measured	140	135	138	155	150	153	225	226	226	284	276	280	298	294	296

1958

1959

1960

1961

1962

1963

1964

1965

1966

Year	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
1958	10	15	20	25	30	35	40	45	50	55	60	65	500
1959	12	18	22	28	32	38	42	48	52	58	62	68	520
1960	14	20	24	30	34	40	44	50	54	60	64	70	540
1961	16	22	26	32	36	42	46	52	56	62	66	72	560
1962	18	24	28	34	38	44	48	54	58	64	68	74	580
1963	20	26	30	36	40	46	50	56	60	66	70	76	600
1964	22	28	32	38	42	48	52	58	62	68	72	78	620
1965	24	30	34	40	44	50	54	60	64	70	74	80	640
1966	26	32	36	42	46	52	56	62	66	72	76	82	660

1958-1966
 1959-1966
 1960-1966
 1961-1966
 1962-1966
 1963-1966
 1964-1966
 1965-1966
 1966-1966

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 23 June 1958Test No. 5Flow Rate: 0.865 #/minPower Input: 41.6 watts/inInlet Air Temp.: 91°F; Reference Junction (Room temp.): 90°F

Time sec	Metal Temp. (°F) x = 6"			Metal Temp. (°F) x = 12"			Metal Temp. (°F) x = 36"			Metal Temp. (°F) x = 60"			Metal Temp. (°F) x = 66"		
	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG
5	37	41	39	40	46	43	57	61	59	32	42	37	50	52	51
10	93	87	90	98	93	96	132	131	132	110	112	111	128	125	127
15	136	126	131	150	148	149	200	202	201	180	175	178	205	193	199
20	173	158	166	203	194	199	267	266	267	243	238	241	268	265	267
30	220	207	214	274	268	271	375	370	373	364	362	363	398	382	390
40	250	237	244	322	322	322	446	444	445	470	463	467	495	478	487
50	270	257	264	356	354	355	502	495	499	455	552	554	568	550	559
70	293	266	280	392	386	389	566	551	559	655	655	655	662	638	650
90	305	261	283	412	402	407	598	581	590	710	703	707	712	690	701
120	308	272	290	418	406	412	615	597	606	740	732	735	732	712	722
150	308	273	290	422	-	-	615	-	-	757	-	-	740	-	-
measured	402	402	402	440	447	444	618	610	614	795	780	788	785	781	783

TABLE 4 (Continued)

RAW DATA - METAL TEMPERATURES

Date: 23 June 1958

Flow Rate: 0.865 #/min

Power Input: 39.9 watts/in

Inlet Air Temp.: 97°F; Reference Junction (Room Temp.): 92°F

Test No. 6

Time Sec	Metal Temp. (°F) x = 6"			Metal Temp. (°F) x = 12"			Metal Temp. (°F) x = 36"			Metal Temp. (°F) x = 60"			Metal Temp. (°F) x = 66"		
	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG	Run 1	Run 2	AVG
5	42	33	38	31	30	31	47	42	45	25	32	29	45	42	44
10	80	73	77	67	70	69	97	97	97	75	90	82	95	93	94
15	120	121	121	111	110	111	157	150	154	128	148	138	147	156	152
20	152	146	149	148	145	147	210	202	206	197	193	195	212	202	207
30	195	188	192	200	200	200	282	276	279	266	275	270	300	294	297
40	218	222	220	230	239	235	335	334	335	338	350	344	365	362	364
50	237	237	237	257	264	261	382	376	379	402	396	399	432	412	422
70	251	256	254	286	293	290	429	426	428	473	485	479	512	493	503
90	252	266	259	298	307	303	445	460	448	508	538	523	542	550	546
120	256	268	262	300	313	307	453	466	460	518	563	541	542	568	555
150	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
measured	310	317	315	345	350	348	477	482	480	610	613	612	620	621	621

TABLE 5

RAW DATA - FLUID EXIT TEMPERATURE

Refer to Table 4 for Test Conditions

Time sec	Test No. 1			Test No. 2			Test No. 3		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	55	50	53	39	48	44	50	46	48
10	85	102	94	104	103	104	105	103	104
15	145	148	147	160	155	158	152	168	160
20	187	189	188	206	201	204	200	200	200
30	266	266	266	288	280	284	294	280	287
40	327	322	325	347	345	346	372	355	364
50	373	363	368	402	392	397	440	418	429
70	438	418	428	468	462	465	539	507	523
90	475	453	465	507	505	506	591	564	578
120	490	480	485	542	537	540	642	623	633
150	500	490	495	555	552	554	668	655	662
measured	503	510	507	568	569	569	690	685	688

STATE OF TEXAS

COMMISSIONERS OF THE GENERAL LAND OFFICE

Section	Range	County	Acres	Original Patent	Original Patentee	Original Patent Date	Original Patent No.
36	10	Wichita	3600	1856	John A. King	1856	1000
35	10	Wichita	3600	1856	John A. King	1856	1000
34	10	Wichita	3600	1856	John A. King	1856	1000
33	10	Wichita	3600	1856	John A. King	1856	1000
32	10	Wichita	3600	1856	John A. King	1856	1000
31	10	Wichita	3600	1856	John A. King	1856	1000
30	10	Wichita	3600	1856	John A. King	1856	1000
29	10	Wichita	3600	1856	John A. King	1856	1000
28	10	Wichita	3600	1856	John A. King	1856	1000
27	10	Wichita	3600	1856	John A. King	1856	1000
26	10	Wichita	3600	1856	John A. King	1856	1000
25	10	Wichita	3600	1856	John A. King	1856	1000
24	10	Wichita	3600	1856	John A. King	1856	1000
23	10	Wichita	3600	1856	John A. King	1856	1000
22	10	Wichita	3600	1856	John A. King	1856	1000
21	10	Wichita	3600	1856	John A. King	1856	1000
20	10	Wichita	3600	1856	John A. King	1856	1000
19	10	Wichita	3600	1856	John A. King	1856	1000
18	10	Wichita	3600	1856	John A. King	1856	1000
17	10	Wichita	3600	1856	John A. King	1856	1000
16	10	Wichita	3600	1856	John A. King	1856	1000
15	10	Wichita	3600	1856	John A. King	1856	1000
14	10	Wichita	3600	1856	John A. King	1856	1000
13	10	Wichita	3600	1856	John A. King	1856	1000
12	10	Wichita	3600	1856	John A. King	1856	1000
11	10	Wichita	3600	1856	John A. King	1856	1000
10	10	Wichita	3600	1856	John A. King	1856	1000
9	10	Wichita	3600	1856	John A. King	1856	1000
8	10	Wichita	3600	1856	John A. King	1856	1000
7	10	Wichita	3600	1856	John A. King	1856	1000
6	10	Wichita	3600	1856	John A. King	1856	1000
5	10	Wichita	3600	1856	John A. King	1856	1000
4	10	Wichita	3600	1856	John A. King	1856	1000
3	10	Wichita	3600	1856	John A. King	1856	1000
2	10	Wichita	3600	1856	John A. King	1856	1000
1	10	Wichita	3600	1856	John A. King	1856	1000

TABLE 5 (Continued)

RAW DATA - FLUID EXIT TEMPERATURE

Refer to Table 4 for Test Conditions

Time sec	Test No. 4			Test No. 5			Test No. 6		
	Run 1	Run 2	Avg	Run 1	Run 2	Avg	Run 1	Run 2	Avg
5	25	25	25	72	73	73	58	56	57
10	47	48	48	143	144	144	120	117	119
15	68	69	69	206	206	206	172	172	172
20	80	89	84	265	264	265	218	222	220
30	122	126	124	362	360	361	287	290	289
40	148	156	152	432	437	435	350	353	352
50	168	175	172	488	488	488	396	402	399
70	194	196	195	552	564	558	455	466	461
90	210	212	211	592	600	596	488	521	505
120	224	226	225	618	632	625	-	-	-
150	228	-	-	632	642	635	-	-	-
measured	240	241	241	660	640	650	518	518	507

TABLE 6
 RESULTS - METAL TEMPERATURES
 Test No. 1 and 6 consolidated

Dimensionless Time, T	X = 0.425 x = 12"			X = 1.275 x = 36"			X = 2.125 x = 60"		
	Smoothed Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M
110	36	0.2	0.156	40	0.3	0.174	43	0.3	0.187
220	81	0.6	0.356	90	0.7	0.396	98	0.7	0.432
440	161	1.4	0.725	189	1.7	0.861	210	1.9	0.963
880	244	2.4	1.14	313	3.2	1.51	368	4.0	1.82
1545	312	3.2	1.50	442	5.3	2.38	511	6.7	2.81
1990	326	3.4	1.57	466	5.8	2.47	548	7.5	3.11
2650	334	3.5	1.62	490	6.2	2.64	585	8.3	3.44
3310	338	3.6	1.65	495	6.3	2.67	594	8.5	3.53

STATE OF TEXAS

COMMISSIONERS OF THE GENERAL LAND OFFICE

REPORT OF THE COMMISSIONERS FOR THE YEAR 1892

Date of Sale	Section	County	Acres	Value	Total	Per Cent	Total	Per Cent
1892	1	1	100	100	100	100	100	100
1893	2	2	200	200	300	100	300	100
1894	3	3	300	300	600	100	600	100
1895	4	4	400	400	1000	100	1000	100
1896	5	5	500	500	1500	100	1500	100
1897	6	6	600	600	2100	100	2100	100
1898	7	7	700	700	2800	100	2800	100
1899	8	8	800	800	3600	100	3600	100
1900	9	9	900	900	4500	100	4500	100
1901	10	10	1000	1000	5500	100	5500	100
1902	11	11	1100	1100	6600	100	6600	100
1903	12	12	1200	1200	7800	100	7800	100
1904	13	13	1300	1300	9100	100	9100	100
1905	14	14	1400	1400	10500	100	10500	100
1906	15	15	1500	1500	12000	100	12000	100
1907	16	16	1600	1600	13600	100	13600	100
1908	17	17	1700	1700	15300	100	15300	100
1909	18	18	1800	1800	17100	100	17100	100
1910	19	19	1900	1900	19000	100	19000	100
1911	20	20	2000	2000	21000	100	21000	100
1912	21	21	2100	2100	23100	100	23100	100
1913	22	22	2200	2200	25300	100	25300	100
1914	23	23	2300	2300	27600	100	27600	100
1915	24	24	2400	2400	30000	100	30000	100
1916	25	25	2500	2500	32500	100	32500	100
1917	26	26	2600	2600	35100	100	35100	100
1918	27	27	2700	2700	37800	100	37800	100
1919	28	28	2800	2800	40600	100	40600	100
1920	29	29	2900	2900	43500	100	43500	100
1921	30	30	3000	3000	46500	100	46500	100
1922	31	31	3100	3100	49600	100	49600	100
1923	32	32	3200	3200	52800	100	52800	100
1924	33	33	3300	3300	56100	100	56100	100
1925	34	34	3400	3400	59500	100	59500	100
1926	35	35	3500	3500	63000	100	63000	100
1927	36	36	3600	3600	66600	100	66600	100
1928	37	37	3700	3700	70300	100	70300	100
1929	38	38	3800	3800	74100	100	74100	100
1930	39	39	3900	3900	78000	100	78000	100
1931	40	40	4000	4000	82000	100	82000	100
1932	41	41	4100	4100	86100	100	86100	100
1933	42	42	4200	4200	90300	100	90300	100
1934	43	43	4300	4300	94600	100	94600	100
1935	44	44	4400	4400	99000	100	99000	100
1936	45	45	4500	4500	103500	100	103500	100
1937	46	46	4600	4600	108100	100	108100	100
1938	47	47	4700	4700	112800	100	112800	100
1939	48	48	4800	4800	117600	100	117600	100
1940	49	49	4900	4900	122500	100	122500	100
1941	50	50	5000	5000	127500	100	127500	100
1942	51	51	5100	5100	132600	100	132600	100
1943	52	52	5200	5200	137800	100	137800	100
1944	53	53	5300	5300	143100	100	143100	100
1945	54	54	5400	5400	148500	100	148500	100
1946	55	55	5500	5500	154000	100	154000	100
1947	56	56	5600	5600	159600	100	159600	100
1948	57	57	5700	5700	165300	100	165300	100
1949	58	58	5800	5800	171100	100	171100	100
1950	59	59	5900	5900	177000	100	177000	100
1951	60	60	6000	6000	183000	100	183000	100
1952	61	61	6100	6100	189100	100	189100	100
1953	62	62	6200	6200	195300	100	195300	100
1954	63	63	6300	6300	201600	100	201600	100
1955	64	64	6400	6400	208100	100	208100	100
1956	65	65	6500	6500	214700	100	214700	100
1957	66	66	6600	6600	221400	100	221400	100
1958	67	67	6700	6700	228200	100	228200	100
1959	68	68	6800	6800	235100	100	235100	100
1960	69	69	6900	6900	242100	100	242100	100
1961	70	70	7000	7000	249200	100	249200	100
1962	71	71	7100	7100	256400	100	256400	100
1963	72	72	7200	7200	263700	100	263700	100
1964	73	73	7300	7300	271100	100	271100	100
1965	74	74	7400	7400	278600	100	278600	100
1966	75	75	7500	7500	286200	100	286200	100
1967	76	76	7600	7600	293900	100	293900	100
1968	77	77	7700	7700	301700	100	301700	100
1969	78	78	7800	7800	309600	100	309600	100
1970	79	79	7900	7900	317600	100	317600	100
1971	80	80	8000	8000	325700	100	325700	100
1972	81	81	8100	8100	333900	100	333900	100
1973	82	82	8200	8200	342200	100	342200	100
1974	83	83	8300	8300	350600	100	350600	100
1975	84	84	8400	8400	359100	100	359100	100
1976	85	85	8500	8500	367700	100	367700	100
1977	86	86	8600	8600	376400	100	376400	100
1978	87	87	8700	8700	385200	100	385200	100
1979	88	88	8800	8800	394100	100	394100	100
1980	89	89	8900	8900	403100	100	403100	100
1981	90	90	9000	9000	412200	100	412200	100
1982	91	91	9100	9100	421400	100	421400	100
1983	92	92	9200	9200	430700	100	430700	100
1984	93	93	9300	9300	440100	100	440100	100
1985	94	94	9400	9400	449600	100	449600	100
1986	95	95	9500	9500	459200	100	459200	100
1987	96	96	9600	9600	468900	100	468900	100
1988	97	97	9700	9700	478700	100	478700	100
1989	98	98	9800	9800	488600	100	488600	100
1990	99	99	9900	9900	498600	100	498600	100
1991	100	100	10000	10000	508700	100	508700	100

Total Land Sold

Total Value of Land Sold

TABLE 6 (Continued)

RESULTS - METAL TEMPERATURES

Test No. 2

Non-Dimensional Time, T	X = 0.440 x = 12"			X = 1.32 x = 36"			X = 2.20 x = 60"		
	Smoother Temp., OF	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smoother Temp., OF	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smoother Temp., OF	External Heat Loss, (watts/in)	Non-Dimensional Temp., M
94	35	0.2	0.13	41	0.3	0.15	47	0.3	0.18
187	82	0.6	0.31	91	0.7	0.34	99	0.7	0.37
374	180	1.6	0.70	204	1.9	0.80	220	2.0	0.87
748	287	2.9	1.17	353	3.9	1.48	394	4.5	1.70
1310	353	3.8	1.47	467	5.8	2.22	550	7.5	2.68
1680	370	4.1	1.57	500	6.4	2.33	660	8.8	3.12
2240	380	4.2	1.62	523	6.9	2.48	628	9.4	3.33
2800	382	4.2	1.63	538	7.3	2.60	645	9.8	3.48

TABLE 1. WATER TEMPERATURES

Cont No. 2

Date	Time	Depth (m)	Temperature (°C)		Depth (m)	Temperature (°C)		Date	Time
			Surface	Bottom		Surface	Bottom		
1961	0800	0	18.0	18.0	0	18.0	18.0	1961	0800
		1	17.0	17.0	1	17.0	17.0		
		2	16.0	16.0	2	16.0	16.0		
		3	15.0	15.0	3	15.0	15.0		
		4	14.0	14.0	4	14.0	14.0		
		5	13.0	13.0	5	13.0	13.0		
		6	12.0	12.0	6	12.0	12.0		
		7	11.0	11.0	7	11.0	11.0		
		8	10.0	10.0	8	10.0	10.0		
		9	9.0	9.0	9	9.0	9.0		
		10	8.0	8.0	10	8.0	8.0		
		11	7.0	7.0	11	7.0	7.0		
		12	6.0	6.0	12	6.0	6.0		
		13	5.0	5.0	13	5.0	5.0		
		14	4.0	4.0	14	4.0	4.0		
		15	3.0	3.0	15	3.0	3.0		
		16	2.0	2.0	16	2.0	2.0		
		17	1.0	1.0	17	1.0	1.0		
		18	0.0	0.0	18	0.0	0.0		

TABLE 6 (Continued)

RESULTS - METAL TEMPERATURES

Test No. 3

Non-Dimensional Time, T	X = 0.474 x = 12"			X = 1.42 x = 36"			X = 2.37 x = 60"		
	Smootherd Temp., °F	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smootherd Temp., °F	External Heat Loss, (watts/in)	Non-Dimensional Temp., M	Smootherd Temp., °F	External Heat Loss, (watts/in)	Non-Dimensional Temp., M
71	37	0.2	0.10	40	0.3	0.11	44	0.3	0.13
142	77	0.5	0.22	88	0.6	0.25	98	0.7	0.28
284	157	1.3	0.46	188	1.6	0.56	210	1.9	0.63
568	269	2.6	0.82	342	3.7	1.08	387	4.3	1.26
993	360	3.9	1.15	508	6.6	1.80	600	8.7	2.34
1280	403	4.6	1.33	576	8.1	2.18	678	10.7	2.91
1700	423	4.9	1.41	617	9.1	2.45	730	13.0	3.52
2130	432	5.1	1.45	645	9.8	2.44	767	13.5	3.82

REPORT OF THE

Year	Month	Day	Hour	Wind	Temp	Barometer		Direction	Remarks
						At Sea	On Shore		
1880	Jan	1	10	10	50	30.0	30.0	W	17
1880	Jan	2	10	10	50	30.0	30.0	W	18
1880	Jan	3	10	10	50	30.0	30.0	W	19
1880	Jan	4	10	10	50	30.0	30.0	W	20
1880	Jan	5	10	10	50	30.0	30.0	W	21
1880	Jan	6	10	10	50	30.0	30.0	W	22
1880	Jan	7	10	10	50	30.0	30.0	W	23
1880	Jan	8	10	10	50	30.0	30.0	W	24
1880	Jan	9	10	10	50	30.0	30.0	W	25
1880	Jan	10	10	10	50	30.0	30.0	W	26
1880	Jan	11	10	10	50	30.0	30.0	W	27
1880	Jan	12	10	10	50	30.0	30.0	W	28
1880	Jan	13	10	10	50	30.0	30.0	W	29
1880	Jan	14	10	10	50	30.0	30.0	W	30

TABLE 6 (Continued)
RESULTS - METAL TEMPERATURES

Test No. 4

Dimensionless Time, T	X = 0.425 x = 12"			X = 1.275 x = 36"			X = 2.125 x = 60"		
	Smoother Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoother Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoother Temp. (°F)	External Heat Loss (watts/in)	Dimensionless Temp., M
110	20	0.1	0.19	25	0.1	0.24	30	0.2	0.29
220	36	0.2	0.35	46	0.3	0.45	51	0.4	0.68
440	70	0.5	0.70	89	0.6	0.89	105	0.7	1.06
880	112	0.8	1.13	156	1.3	1.64	189	1.7	2.08
1545	140	1.2	1.46	204	1.8	2.24	252	2.4	2.91
1990	150	1.2	1.57	219	2.0	2.45	270	2.6	3.18
2650	153	1.3	1.61	229	2.1	2.58	282	2.8	3.38
3310	154	1.3	1.62	235	2.2	2.67	287	2.9	3.47

1910
 STATE OF CALIFORNIA
 DEPARTMENT OF AGRICULTURE
 BUREAU OF PLANT INDUSTRY

No.	Name of Plant	Origin	Date of Introduction	Cultivated	Naturalized	Remarks	Authority
1	Apple	Europe	1792	Y			Gray
2	Orange	Spain	1549	Y			Gray
3	Lemon	Spain	1549	Y			Gray
4	Pineapple	Caribbean	1493	Y			Gray
5	Guava	Caribbean	1493	Y			Gray
6	Avocado	Central America	1519	Y			Gray
7	Strawberry	Europe	1792	Y			Gray
8	Raspberry	Europe	1792	Y			Gray
9	Blackberry	Europe	1792	Y			Gray
10	Blueberry	Europe	1792	Y			Gray
11	Cherry	Europe	1792	Y			Gray
12	Peach	Europe	1792	Y			Gray
13	Plum	Europe	1792	Y			Gray
14	Apricot	Europe	1792	Y			Gray
15	Almond	Europe	1792	Y			Gray
16	Walnut	Europe	1792	Y			Gray
17	Chestnut	Europe	1792	Y			Gray
18	Walrus	Europe	1792	Y			Gray
19	Fig	Europe	1792	Y			Gray
20	Pistachio	Europe	1792	Y			Gray
21	Macadamia	South Africa	1840	Y			Gray
22	Coconut	Polynesia	1792	Y			Gray
23	Pineapple	Caribbean	1493	Y			Gray
24	Guava	Caribbean	1493	Y			Gray
25	Avocado	Central America	1519	Y			Gray
26	Strawberry	Europe	1792	Y			Gray
27	Raspberry	Europe	1792	Y			Gray
28	Blackberry	Europe	1792	Y			Gray
29	Blueberry	Europe	1792	Y			Gray
30	Cherry	Europe	1792	Y			Gray
31	Peach	Europe	1792	Y			Gray
32	Plum	Europe	1792	Y			Gray
33	Apricot	Europe	1792	Y			Gray
34	Almond	Europe	1792	Y			Gray
35	Walnut	Europe	1792	Y			Gray
36	Chestnut	Europe	1792	Y			Gray
37	Walrus	Europe	1792	Y			Gray
38	Fig	Europe	1792	Y			Gray
39	Pistachio	Europe	1792	Y			Gray
40	Macadamia	South Africa	1840	Y			Gray

TABLE 6 (Continued)
RESULTS - METAL TEMPERATURES

Test No. 5

Dimensionless Time, T	X = 0.425 x = 12"			X = 1.275 x = 36"			X = 2.125 x = 60"		
	Smoothed Temp., OF	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp., (OF)	External Heat Loss (watts/in)	Dimensionless Temp., M	Smoothed Temp. (OF)	External Heat Loss, (watts/in)	Dimensionless Temp., M
110	42	0.3	0.14	45	0.3	0.15	48	0.3	0.15
220	97	0.7	0.32	119	0.9	0.39	130	1.1	0.43
440	188	1.6	0.63	229	2.2	0.77	255	2.5	0.87
880	318	3.3	1.10	420	4.9	1.52	472	5.9	1.76
1545	388	4.4	1.39	535	7.2	2.07	635	9.5	2.63
1990	407	4.7	1.47	575	8.1	2.28	696	10.8	3.00
2650	415	4.8	1.50	593	8.5	2.38	715	12.0	3.21
3310	421	4.9	1.53	606	8.8	2.45	732	12.5	3.35

TABLE 7

RESULTS - FLUID TEMPERATURES

Average data from Test 1 and 6

Dimensionless Time, T	Averaged Temp. ($^{\circ}$ F)	External Heat Loss (watts/in)	Dimensionless Temp., F
110	55	0.3	0.238
220	107	0.8	0.473
330	159	1.3	0.715
440	204	2.0	0.94
660	278	3.0	1.32
880	339	4.3	1.69
1100	384	5.4	2.00
1540	445	7.0	2.50
1980	485	8.0	2.82
2640	508	8.8	3.05
measured	513	9.0	3.11

TEMPERATURE RECORD

Station No. 1000

Time	Temperature (°C)	Remarks	Time	Temperature (°C)
08:00	15.0		14:00	18.0
09:00	16.0		15:00	19.0
10:00	17.0		16:00	20.0
11:00	18.0		17:00	21.0
12:00	19.0		18:00	22.0
13:00	20.0		19:00	23.0
14:00	21.0		20:00	24.0
15:00	22.0		21:00	25.0
16:00	23.0		22:00	26.0
17:00	24.0		23:00	27.0
18:00	25.0		24:00	28.0
19:00	26.0		25:00	29.0
20:00	27.0		26:00	30.0
21:00	28.0		27:00	31.0
22:00	29.0		28:00	32.0
23:00	30.0		29:00	33.0
00:00	31.0		30:00	34.0

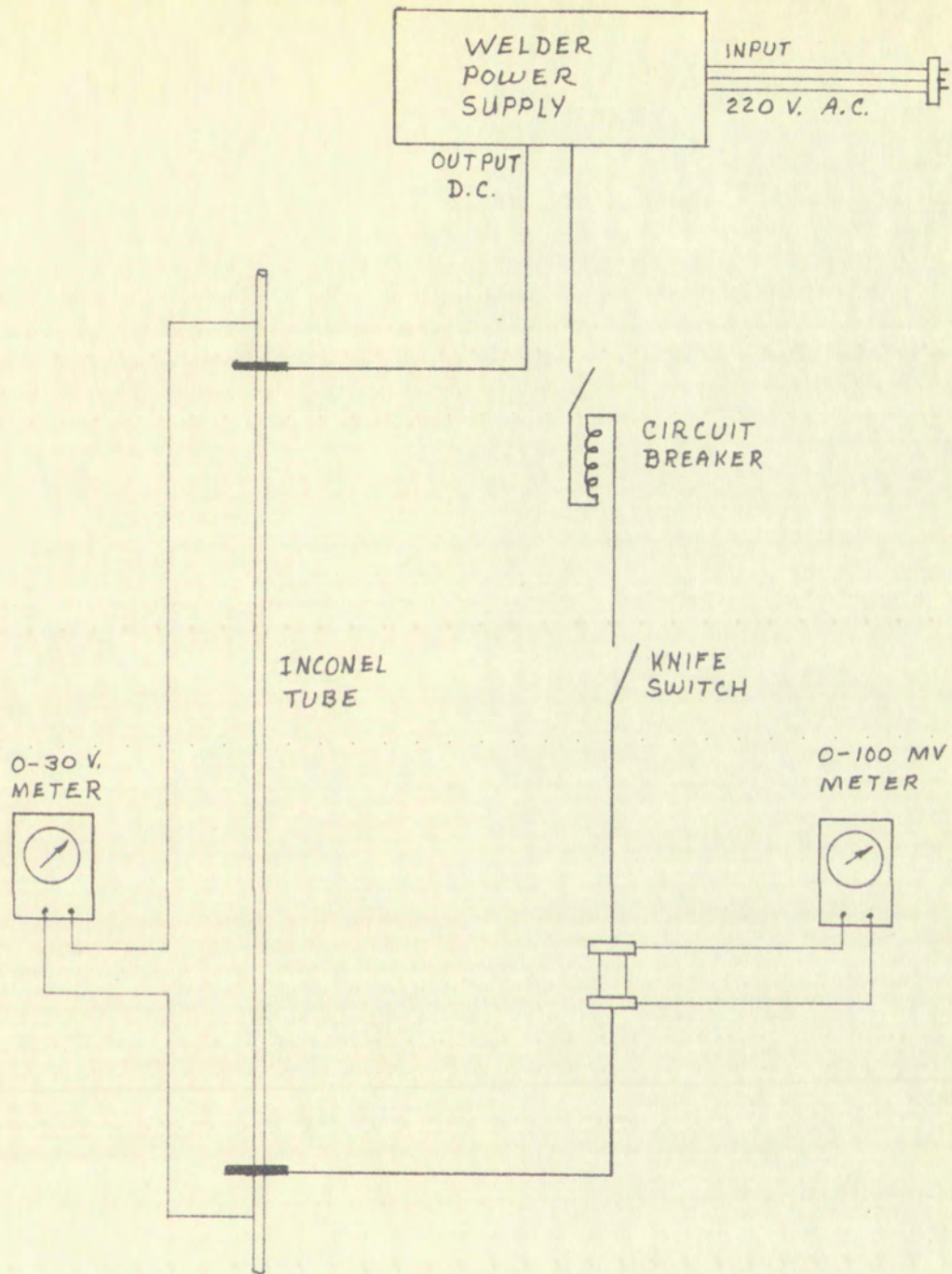
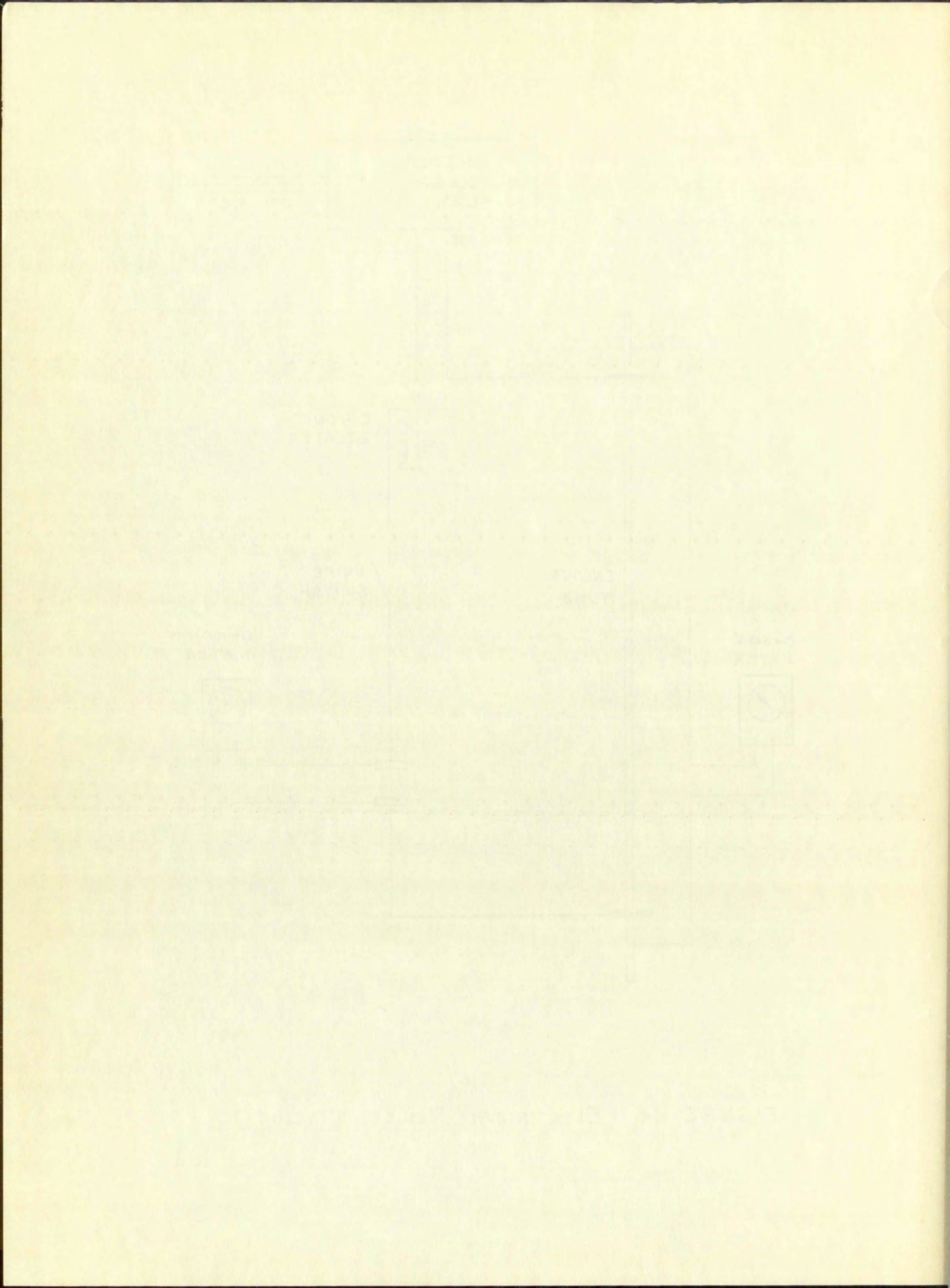


FIGURE 14 ELECTRICAL POWER CIRCUIT



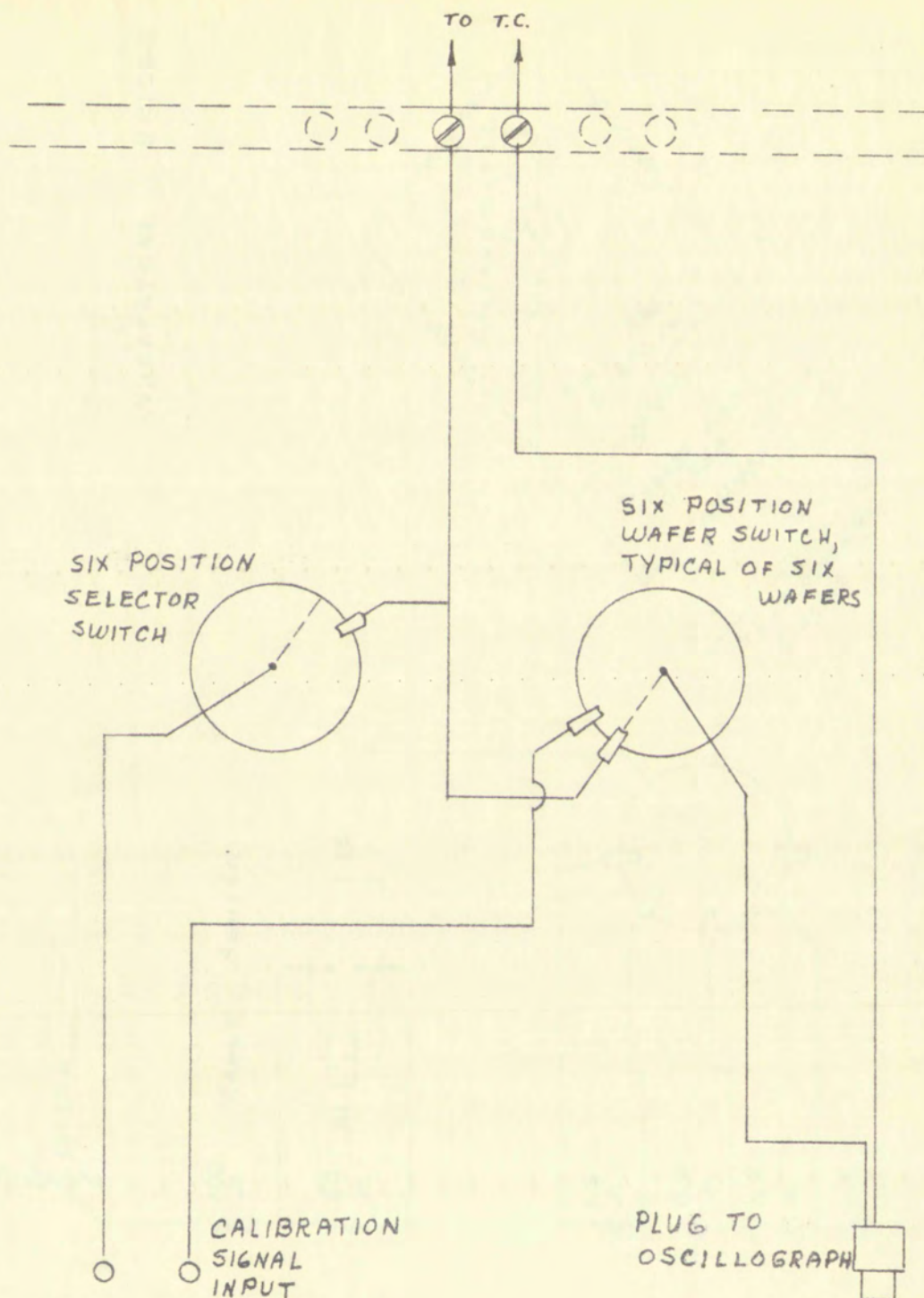
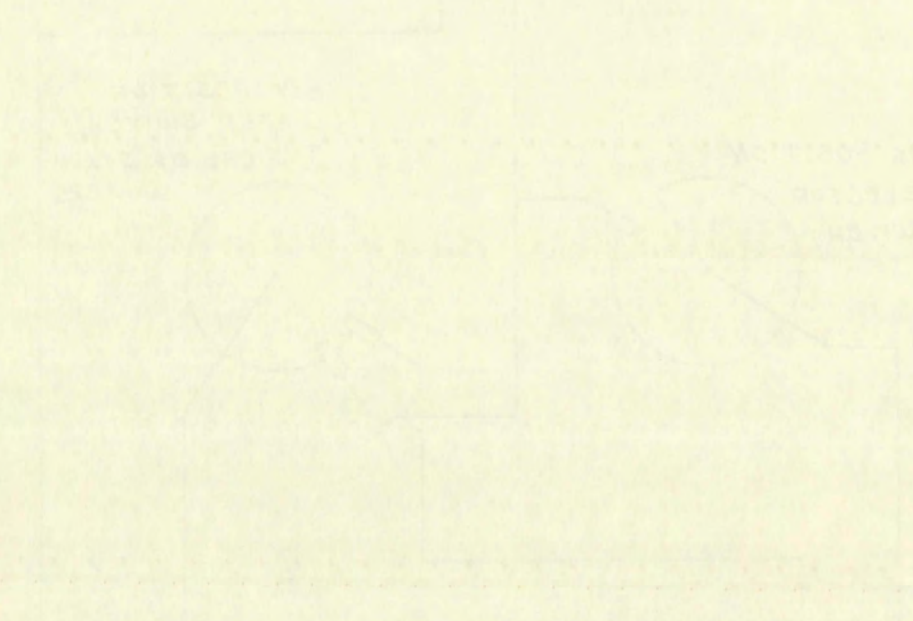


FIGURE 15 TYPICAL PORTION OF T.C. SWITCHING CIRCUIT

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Faint text located below the graph, possibly a legend or a description of the data.

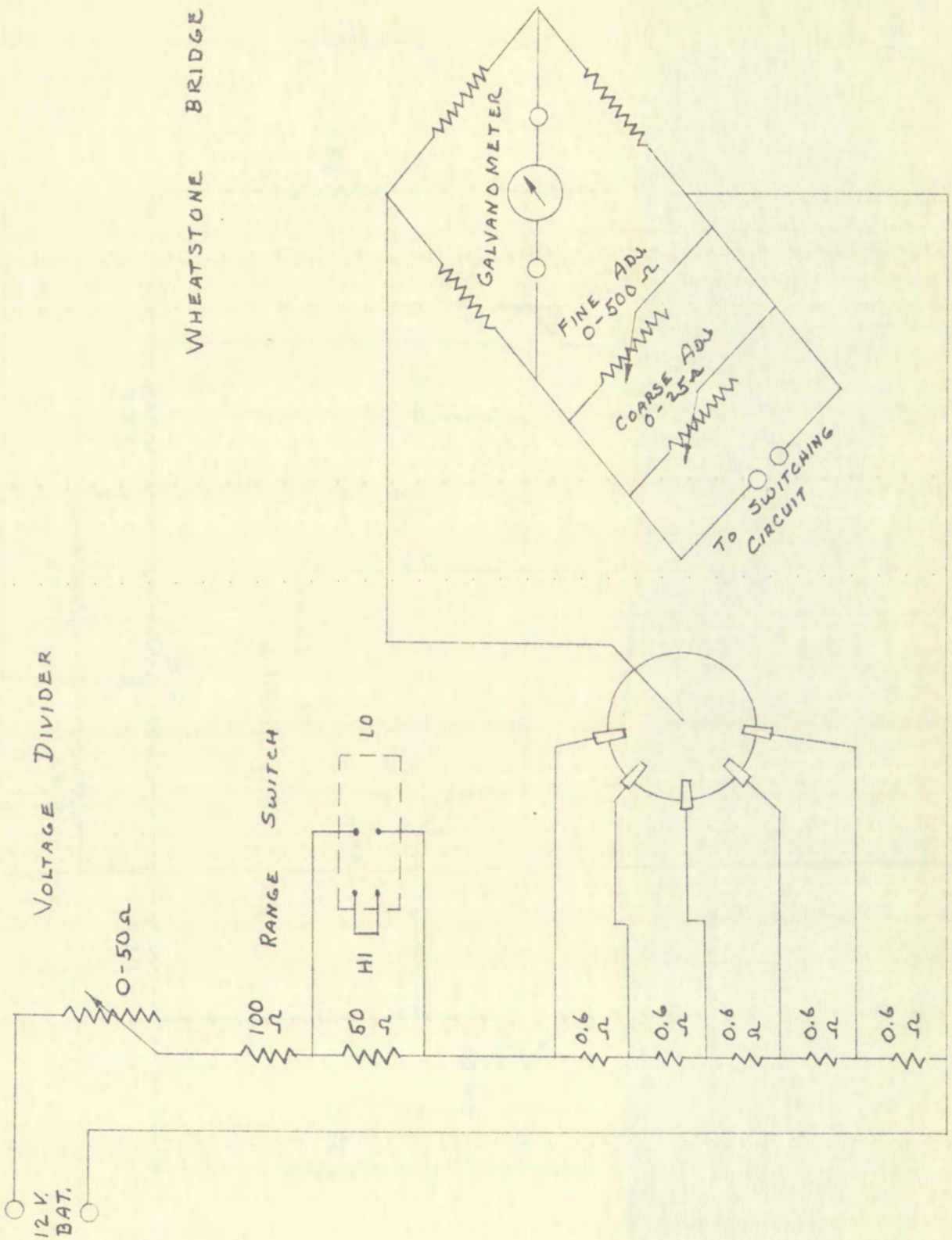
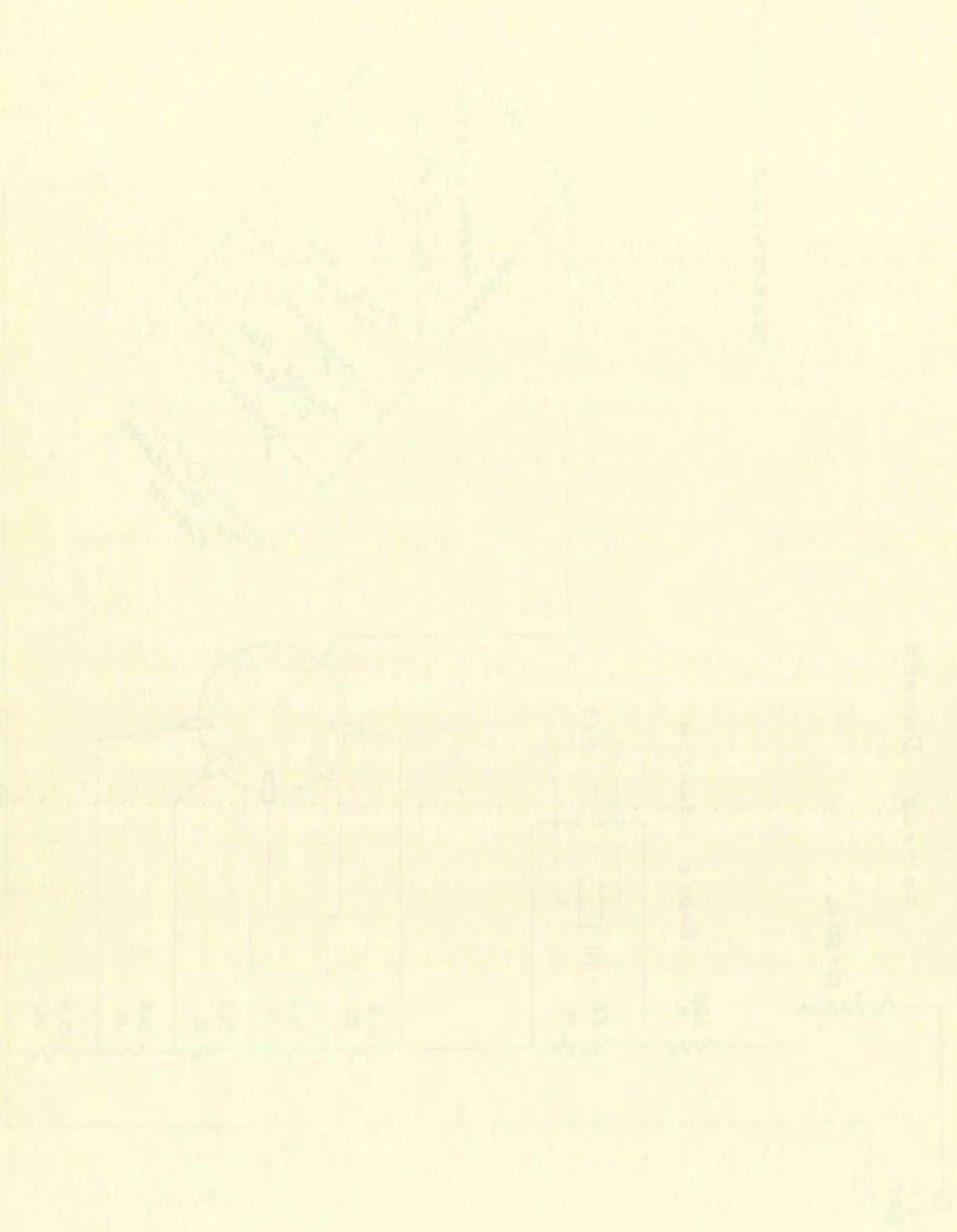


FIGURE 16 CALIBRATION CIRCUIT SCHEMATIC

LICENCE IC Calibration - Circuit Diagram



APPROXIMATE

APPROXIMATE

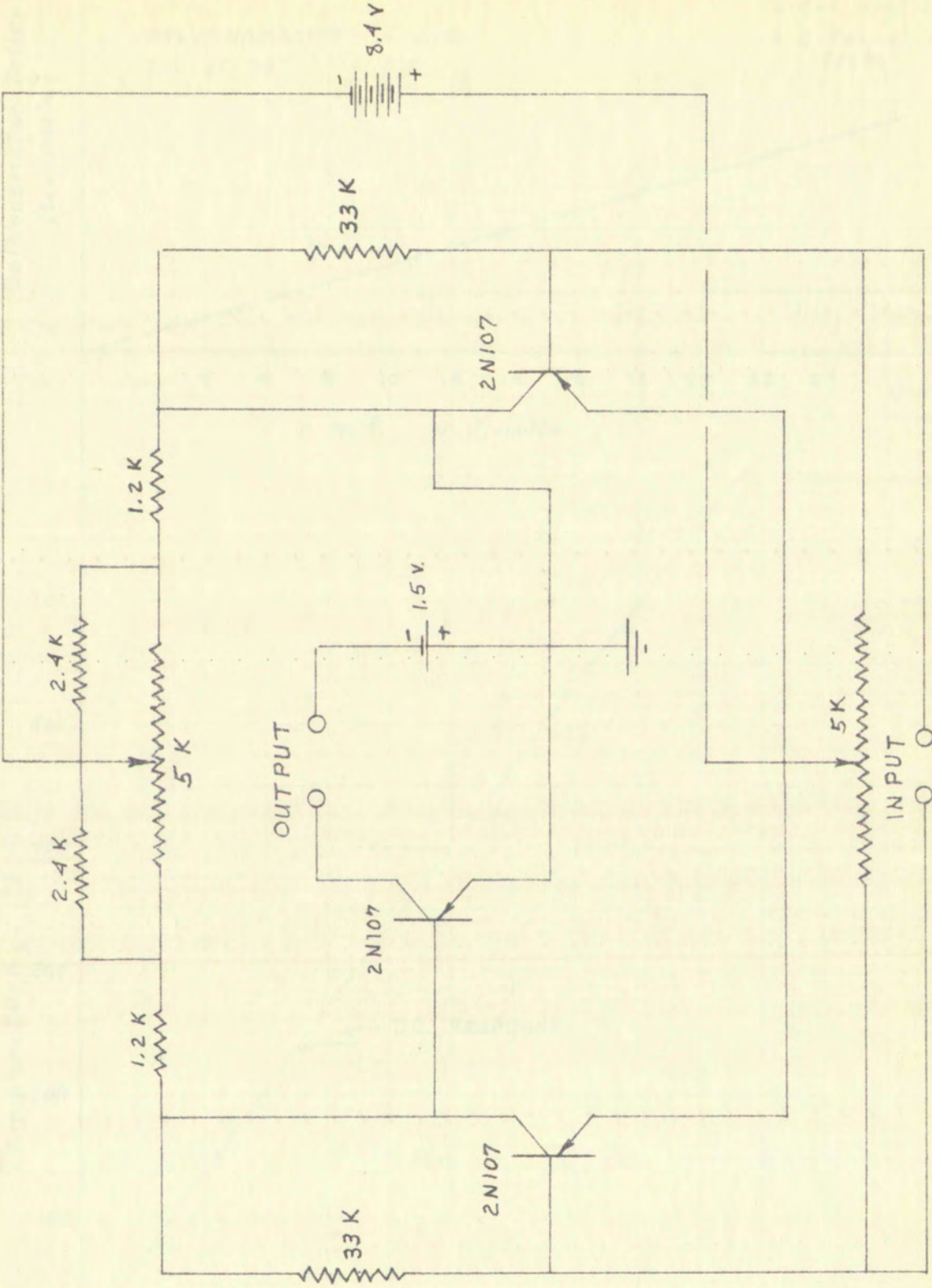
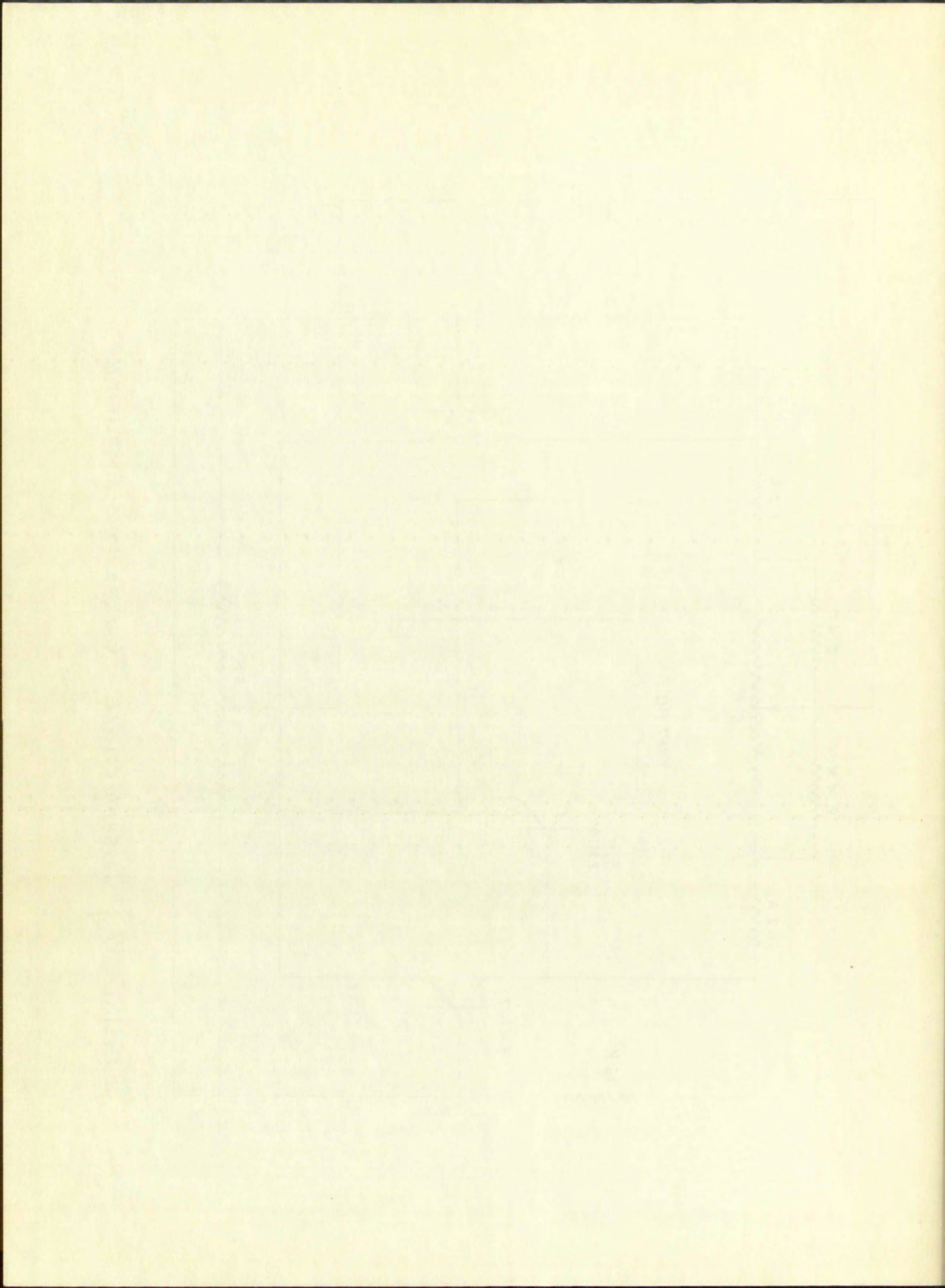


FIGURE 17 THERMOCOUPLE CURRENT AMPLIFIER



Galvanometer Deflection
(too scale)

GALVANOMETER #1 with
T.C. at 36"

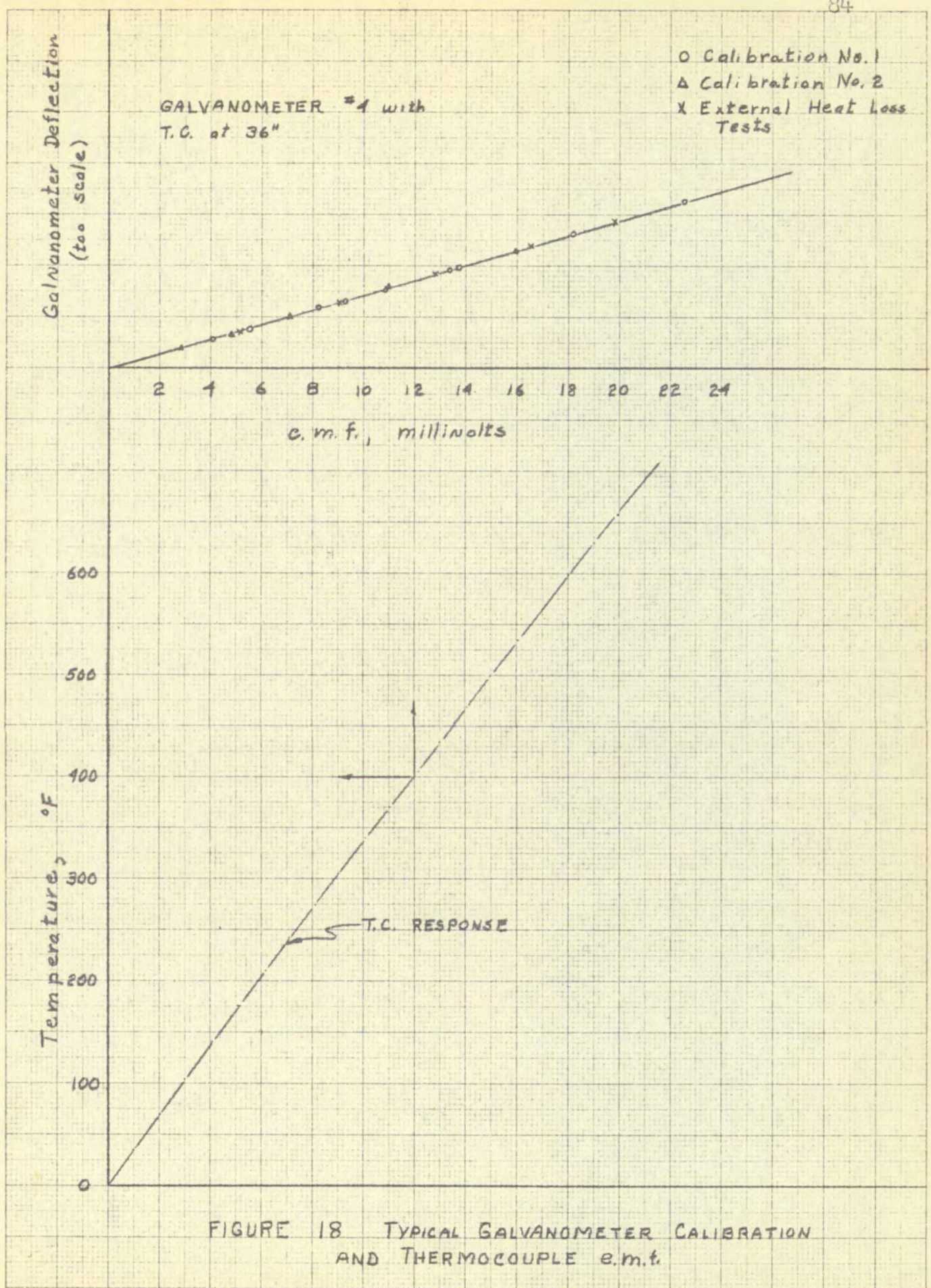
O Calibration No. 1
Δ Calibration No. 2
X External Heat Loss
Tests

2 4 6 8 10 12 14 16 18 20 22 24
c. m. f., millivolts

600
500
400
300
200
100
0
Temperature, °F

T.C. RESPONSE

FIGURE 18 TYPICAL GALVANOMETER CALIBRATION AND THERMOCOUPLE e.m.f.



1000 (1000)

500

100

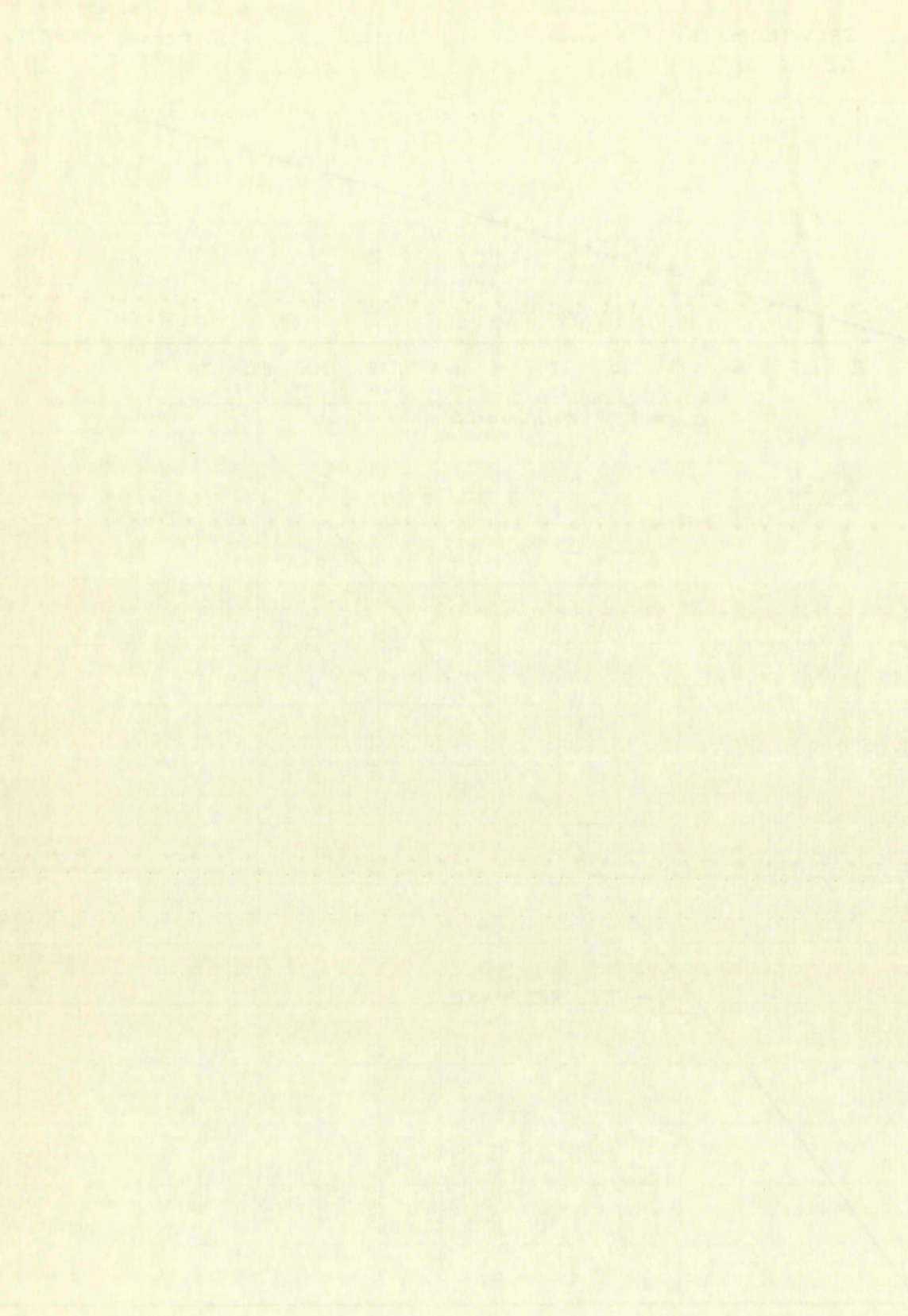
0

1000 (1000)

500

100

0



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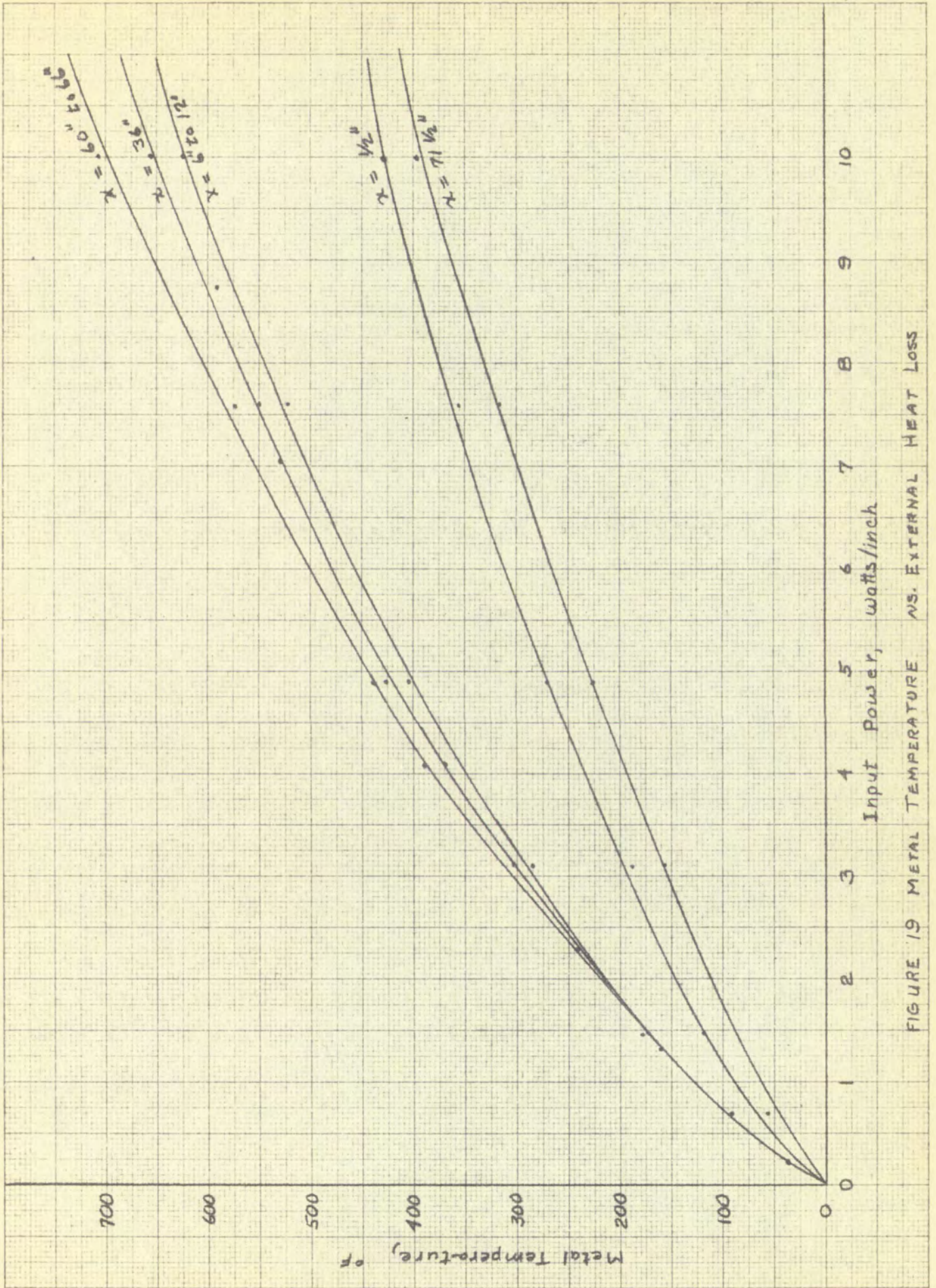
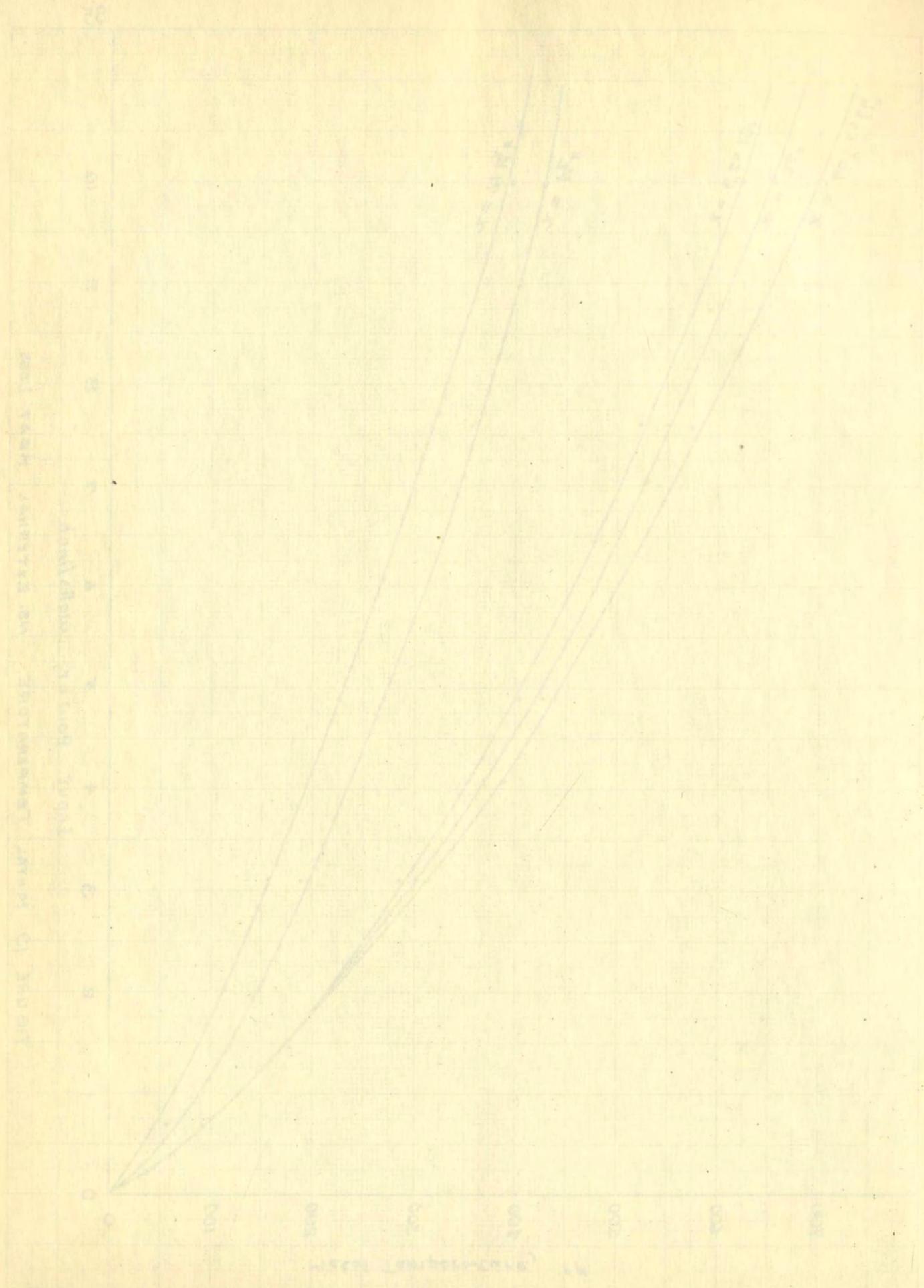
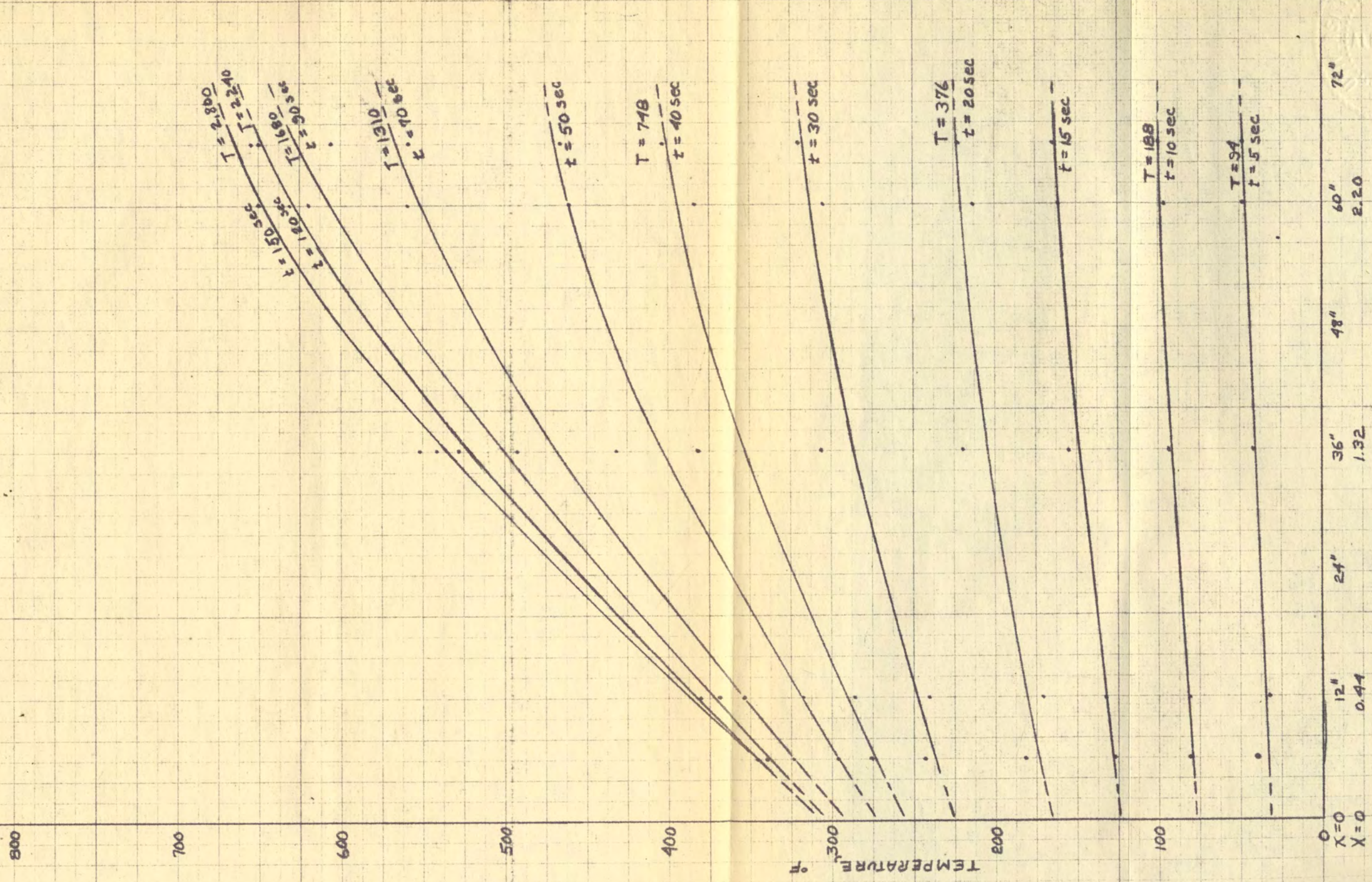


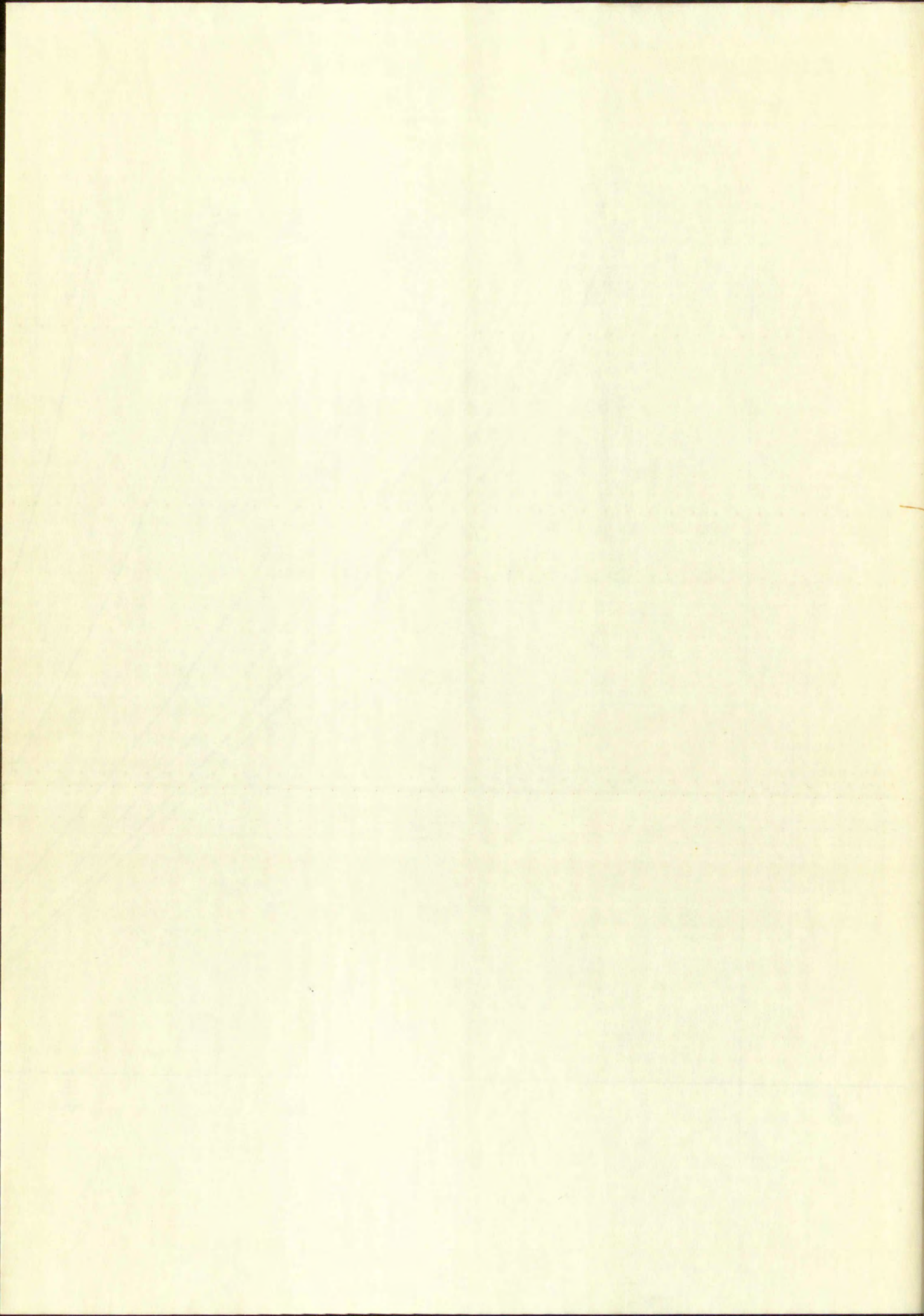
FIGURE 19 METAL TEMPERATURE VS. EXTERNAL HEAT LOSS





DISTANCE DOWNSTREAM

FIGURE 20 DATA SMOOTHING FOR TEST 2



Wesman
Elastic Bond

25% COTTON FIBRE

W. H. B. & Co.
Baltimore, Md.
BY COTTON FIBRE

